Multiwavelet and Biological Signal Processing

Morteza Moazami-Goudarzi, Ali Taheri, Mohammad Pooyan, Reza Mahboobi

***Abstract*—**In this paper we are to find the optimum multiwavelet for compression of electrocardiogram (ECG) signals and then, selecting it for using with SPIHT codec. At present, it is not well known which multiwavelet is the best choice for optimum compression of ECG. In this work, we examine different multiwavelets on 24 sets of ECG data with entirely different characteristics, selected from MIT-BIH database. For assessing the functionality of the different multiwavelets in compressing ECG signals, in addition to known factors such as Compression Ratio (CR), Percent Root Difference (PRD), Distortion (D), Root Mean Square Error (RMSE) in compression literature, we also employed the Cross Correlation (CC) criterion for studying the morphological relations between the reconstructed and the original ECG signal and Signal to reconstruction Noise Ratio (SNR). The simulation results show that the Cardinal Balanced Multiwavelet (cardbal2) by the means of identity (Id) prefiltering method to be the best effective transformation. After finding the most efficient multiwavelet, we apply SPIHT coding algorithm on the transformed signal by this multiwavelet.

***Keywords*—** ECG compression, Prefiltering, Cardinal Balanced Multiwavelet.

I. Introduction

Compressing biological signals, especially ECG has an important role in diagnosis, taking care of patients and signal transfer through communication lines. Therefore, ECG data compression has been one of the most active research areas in biomedical engineering.

Techniques for ECG compression which have been reported in the literature fall mainly into two categories: (1) direct compression such as Amplitude-Zone-Time Epoch Coding (AZTEC) method, the coordinate reduction time coding system (CORTES), turning point (TP) technique, Scan-Along Polygonal Approximation (SAPA), and the long–term prediction (LTP), differential pulse code modulation (DPCM), (2) transformational methods such as Fourier transform, Walsh Transform, Karhunen-Loeve Transform (KLT) and Wavelet Transform (WT). In most cases, direct methods are superior to transform methods with respect to system simplicity and error. However, transform methods usually achieve higher compression rates and are insensitive to noise contained in original ECG signal [1].

Among the methods mentioned above, wavelet transform is an efficient tool in signal processing aimed at compressing ECG signals, detection of QRS complex, analysis of ventricular late potential, etc. The purpose of this paper is to employ the multiwavelet as an extension of wavelet for ECG compression. The primary results of applying multiwavelets in signal processing [2,3,4,5], compression [4,6,7] and noise elimination [4,8,9] indicate the superiority of multiwavelet to wavelet. By comparing compression results, we choose the one which has the best results. After applying multiwavelet transform on ECG signal, we encode the transform coefficients by SPIHT coding algorithm, which has shown superior results in image compression [16] and wavelet compression of ECG signals [15].

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II. Multiwavelet

A. A Brief History of Multiwavelet

Multiwavelets constitute a new chapter which has been added to wavelet theory in recent years. Recently, much interest has been generated in the study of the multiwavelets, where more than one scaling function and mother wavelet are used to represent a given signal.

The first construction for polynomial multiwavelet was given by Alpert, who used them as a basis for the representation of certain operators. Later, Geronimo, Hardin and Massopust constructed a multi-scaling function with 2 components using fractal interpolation.

In [10], multiwavelets based on Cardinal Hermite splines were constructed. In spite of the many theoretical results on multiwavelet, their successful applications to various problem in signal processing are still limited.

Unlike scalar wavelets, in which Mallat's pyramid algorithm have provided a solution for good signal decomposition and reconstruction, a good framework for the application of the multiwavelet is still not available. Nevertheless, several researchers have proposed method of how to apply a given multiwavelet filter for signal and image decomposition. For example, Xia *et al* [10, 11] have proposed new algorithm to compute multiwavelet transform coefficients by using appropriate pre- and post-filtering filters, and have indicated that the energy compaction for discrete multiwavelet transform may be

better than that obtained using conventional discrete scalar wavelet transforms.

So, based on problems mentioned above, finding a multiwavelet that has the most energy compaction is an important subject in signal compression. Our motivation in this work is to find multiwavelet that has the best energy compaction for different ECG signals.

B. Multi-scaling Functions and Multiwavelets

The concept of multi-resolution analysis can be extended from the scalar case to general dimension $r \in \mathbb{N}$. A vector valued function $\mathbf{\Phi} = \begin{bmatrix} \phi_1 & \phi_2 \cdots \phi_r \end{bmatrix}^T$ belonging to $L^2(\mathbb{R})^r$, $r \in \mathbb{N}$ is called a multi-scaling function if the sequence of closed spaces:

$$V_j = \overline{span\left\{2^{j/2}\phi_i\left(2^j. - k\right) : 1 \le i \le r,\ k \in \mathbb{Z}\right\}} \quad (1)$$

for $j \in \mathbb{Z}$ constitute a multi-resolution analysis (MRA) of multiplicity *r* for $L^2(\mathbb{R})$. The multi-scaling function must satisfy the two-scale dilation equation:

$$\mathbf{\Phi}(t) = \sqrt{2}\sum_k \mathbf{G}_k \Phi(2t - k) \quad (2)$$

Now let W_j denote a complementary space of V_j in V_{j+1}. The vector valued function $\mathbf{\Psi} = \begin{bmatrix} \psi_1 & \psi_2 \cdots \psi_r \end{bmatrix}^T$ such that:

$$W_j = \overline{span\left\{2^{j/2}\psi_i\left(2^j. - k\right) : 1 \le i \le r,\ k \in \mathbb{Z}\right\}} \quad (3)$$

for $j \in \mathbb{Z}$ is called a multiwavelet. Multi-scaling and multiwavelet functions must satisfy the two-scale equation:

$$\mathbf{\Psi}(t) = \sqrt{2}\sum_k \mathbf{H}_k \mathbf{\Phi}(2t - k) \quad (4)$$

where $\mathbf{H}_{k \in Z} \in L^2(\mathbb{Z})^{r \times r}$ is an $r \times r$ matrix of coefficients [4, 9]. The two-scale equation (2) and (4) can be realized as a multi filterbank operating on *r* input data streams and filtering them in two 2*r* output data stream, each of which is down-sampled by a factor two. If we denote by $x(t)$ a given signal and assume that $x(t) \in V_0$, then

$$x(t) = \sqrt{2}\sum_k \mathbf{V}_{0,k}^T \mathbf{\Phi}(t - k) \quad (5)$$

and the scaling coefficient $\mathbf{V}_{1,k}$ of the first level can be considered as a result of lowpass multi-filtering and down-sampling:

$$\mathbf{V}_{1,k} = \sum_m \mathbf{G}_{m-2k}\mathbf{V}_{0,m} \quad (6)$$

Analogously, the first level multiwavelet coefficients $\mathbf{W}_{1,k}$ are obtained using high-pass multi-filtering and down-sampling:

$$\mathbf{W}_{1,k} = \sum_m \mathbf{H}_{m-2k}\mathbf{V}_{0,m} \quad (7)$$

Full multiwavelet decomposition of the signal $x(t)$ can be found by iterative filtering of the scaling coefficient:

$$\mathbf{V}_{j,k} = \sum_m \mathbf{G}_{m-2k}\mathbf{V}_{j-1,m} \quad (8)$$

$$\mathbf{W}_{j,k} = \sum_m \mathbf{H}_{m-2k}\mathbf{V}_{j-1,m} \quad (9)$$

Note that $\mathbf{V}_{j,k}$ and $\mathbf{W}_{j,k}$ are $r \times 1$ column vectors.

C. Multiwavelet in Comparison with Wavelet

The multiwavelet idea originates from the generalization of scalar wavelets; Instead of one scaling function and one wavelet, multiple scaling functions and wavelets are used. This leads to more degree of freedom in constructing wavelets. Therefore opposed to scalar wavelets, properties such as compact support, orthogonality, symmetry, vanishing moments and short support can be gathered simultaneously in multiwavelets, which are fundamental in signal processing [4, 5].

The increase in degree of freedom in multiwavelets is obtained at the expense of replacing scalars with matrices, scalar functions with vector functions and single matrices with block of matrices. However, prefiltering is an essential task which should be performed for any use of multiwavelet in the signal processing [4, 12].

D.Prefiltering of the Data

One of the challenges in realizing multiwavelets is the efficient prefiltering. In the case of scalar wavelets, the given signal data are usually assumed to be the scaling coefficients that are sampled at a certain resolution, and hence, we directly apply multi-resolution decomposition on the given signal. But the same technique can not be employed directly in the multiwavelet setting and some prefiltering has to be performed on the input signal prior to multiwavelet decomposition. The type of the prefiltering employed is critical for the success of the results obtained in application

There could be infinitely many ways to do such prefiltering. There exist well known prefilters in literature [11, 13, 14]. The most obvious way to get second input row is just to repeat the first one and use two identical rows of length *n*.

A different way to get the input rows for the multiwavelet filterbank is to preprocess the given scalar signal *f*(*n*). In our implementation, first we refer to repeated row (rr) and second we refer to approximation prefilter (app).

For the balanced multiwavelet, the identity (ID) prefilter is used. This prefilter just separates the input data in two streams: one consisting of even numbered samples, the other consisting of odd numbered samples.

III. Compression Method

A. Multiwavelet Decomposition

The goal of this section is to apply nine multiwavelets with prefiltering mentioned above on ECG signals. We retain the same number of largest coefficients for each multiwavelet, and then invert the algorithm to reconstruct the signal and measure the performance of each multiwavelet by assessment criteria. All of our tests are applied on the first 2048 samples of the Lead I from MIT-BIH records 100, 101, 102, 103, 104, 105, 106, 107, 118, 119, 200, 201, 202, 203, 205, 207, 208, 209, 210, 212, 214, 215, 217, 219. A simple threshold compression method has been applied based on the following steps:

1) prefiltering and multiwavelet decomposition up to 6 levels.
2) keeping the first *N* largest coefficients of the decomposition.
3) reconstruction from *N* coefficients.

For simplicity, we have considered $N = 125$, corresponding to compression ratios of 16.384 for all signals.

B. Assessment Criteria

An ECG compression algorithm is judged by its ability to minimize the distortion while retaining all significant features of the signal. The distortion in reconstruction has been computed by means of the following formula:

$$\mathrm{D} = \frac{\|x_{or} - x_{re}\|^2}{\|x_{or}\|^2}$$

where x_{or} is the original signal and x_{re} is the reconstructed signal.

Another method that can be used to measure distortion is PRD. So x_{or} and x_{re} are signals of length *N*, PRD can be defined as:

$$\mathrm{PRD} = \sqrt{\frac{\sum (x_{or} - x_{re})^2}{\sum (x_{or})^2}} \times 100\%$$

Another criterion we use for measuring distortion is RMSE. In data compression, we are interested in finding an optimal approximation for minimizing this criterion as defined by the following formula:

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_{or} - x_{re})^2}$$

However, since the similarity between the reconstructed and original signal is very important from the clinical point of view, the CC is employed to evaluate the similarity between the original signal and its reproduced version, defined as:

$$\mathrm{CC} = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}\sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})^2}}$$

where x_i and y_i are the samples of the original signal and its reproduced version, $\bar{x}$ and $\bar{y}$ are their average values, respectively.

Another criterion that is used here is SNR and is given by:

$$\mathrm{SNR} = 20\log\left(\frac{\hat{\sigma}_{x_{or}}}{\hat{\sigma}_{x_{or}-x_{re}}}\right) \qquad \text{(dB)}$$

where $\hat{\sigma}$ denotes the standard deviation estimator. In signal processing we are interested in finding an optimal approximation for maximizing this criterion.

After testing different multiwavelets and assessing their performances in ECG compression (with respect to indexes such as D, PRD, CC,), we choose that which has the best performance and use it along with the SPIHT coding algorithm. The reason for using codec is firstly to obtain more data reduction. But we must do a tradeoff between the complexity of codec and the increase in CR we gain. Secondly, by some coding methods (such as what we use here, SPIHT) we obtain the coded transform coefficients as a simple bit stream, which is very more efficient for transmission, in comparison with *raw* coefficients that should be treated as, for example, 32-bit double numbers.

IV. SPIHT Coding Algorithm

A. Embedded Coding

In this paper we use SPIHT coding algorithm for coding multiwavelet transform coefficients of ECG signal. Set partitioning in hierarchical trees (SPIHT) is an embedded coding technique. In an embedded coding algorithm, all encodings of the same signal at lower bit rates (than target rate) are embedded at the beginning of the bit stream for the target bit rate. So we can use any amount of bits received for decoding, at a lower bit rate that can be achieved when using the whole bit stream of the coded signal. Effectively, bits are ordered in importance. This type of coding is especially useful for progressive transmission and transmission over a noisy channel. Using an embedded code, an encoder can terminate the encoding process at any point, thereby allowing a target rate or distortion parameter to be met exactly. Typically, some target parameters, such as bit count, is monitored in the encoding process and when the target is met, the encoding simply stops. Similarly, given a bit stream, the decoder can cease decoding at any point and can produce reconstruction corresponding to all lower-rate encodings.

Embedded coding is similar in spirit to binary finite precision representations of real numbers. All real numbers can be represented by a string of binary digits. For each digit added to the right, more precision is added. Yet, encoding can cease at any time and provide the best representation of the real number achievable within the framework of the binary digit representation. Similarly,

the embedded coder can cease at any time and provide the best representation of the signal achievable within its framework.

Embedded zerotrees of wavelet (EZW), introduced by J. M. Shapiro [17] is an embedded coding algorithm for image compression. It works on discrete wavelet transform coefficients of an image. It is very effective and computationally simple technique for image compression. SPIHT algorithm introduced for image compression in [16] is a refinement to EZW and uses its principles of operation. These principles are partial ordering of transform coefficients by magnitude with a set partitioning sorting algorithm, ordered bit plane transmission and exploitation of self-similarity across different scales of an image wavelet transform. The partial ordering is done by comparing the transform coefficients magnitudes with a set of octavely decreasing thresholds. In fact, in this algorithm, a transmission priority is assigned to each coefficient to be transmitted. Using these rules, the encoder always transmits the most significant bit to the decoder. SPIHT has even better performance than EZW in image compression. In [15], SPIHT algorithm is modified for 1-D signals and used for wavelet compression of ECG signals, that we call it 1D-SPIHT. We apply it on multiwavelet coefficients of ECG signal.

B. Details of SPIHT Algorithm

Here we explain the details of 1-D SPIHT based on 1D-DWT coefficients of a signal. It's directly applicable for multiwavelet transform coefficients. We directly apply the 1D-SPIHT codec over the subband coefficients from multiwavelet decomposition up to six levels. 1-D SPIHT is one of the excellent embedded coding systems that deserve great attention. In the 1-D SPIHT, the original signal is first decomposed into several subbands by 1D-DWT. Then a special tree structure called the *spatial orientation tree* is defined according to the similarity among coefficients across subbands. Such an orientation tree represents the parent-offspring relationship among these quadrate mirror filters (QMF) decomposition subbands. We use arrows in Fig. 1 to illustrate the parent-offspring relationship defined in the 1D-SPIHT and a 5-level 1D-DWT is assumed. Each black dot in this figure denotes a wavelet coefficient.

In Fig. 1, the subbands are arranged from lowpass subbands to highpass subbands. Let x_i be any wavelet coefficient and i denote its corresponding coordinate. There are three important definitions in the 1D-SPIHT parent-offspring relationship as shown in Fig. 2:

1) $O(i)$: offspring $O(i)$ represents the set of the 2 coefficients (as pointed by arrows) of next higher subband from coefficient x_i.
2) $D(i)$: the descendent $D(i)$ of coefficient x_i is the set containing all offspring in all later subbands.
3) $L(i)$: a set defined by $L(i) = D(i) - O(i)$

We can observe in Fig. 1 that according to this relationship, one coefficient in the lowest frequency subband (i.e. $\mathbf{x}_{L5}$) has no descendent in terms of orientation trees.

In order to exploit the self similarity during the 1D-SPIHT coding process, several oriented trees are taken from a wavelet transformed signal. Every tree is rooted as the corresponding top-most lowpass subband. The 1D-SPIHT algorithm assumes that each coefficient x_i is a good predictor of the coefficients which are represented by the subtree rooted by x_i, i.e. $D(i)$. the overall procedure is controlled by an attribute, which gives information on the significance of the coefficients. A coefficient of the wavelet transformed signal is *significant* with respect to a threshold k if its magnitude is larger than 2^k. Otherwise it's called *insignificant* with respect to the threshold k. It can be described as:

$$S_k(x_i) = \begin{cases} 1, & \text{if } |x_i| \geq 2^k \\ 0, & \text{otherwise} \end{cases}$$

where $S_k(x_i)$ denotes the significance of x_i with respect to a threshold k.

In the 1D-SPIHT, the wavelet coefficients are classified in three sets, namely the list of insignificant points (LIP) which contains the coordinate of those coefficients that are insignificant with respect to the current threshold k, the list of significant points (LSP) which contains the coordinates of those coefficients that are significant with respect to k, and the list of insignificant sets (LIS) which contains the coordinates of the roots of insignificant subtrees. In addition, the contents of LIS are classified in types A and B, which represent the $D(i)$ and $L(i)$ cases, respectively. We use 22 steps to depict the overall 1D-SPIHT coding process as follows:

(0) **Initialization**

Compute and output $k = \left\lfloor \log_2 \max_i |x_i| \right\rfloor$, $0 \leq i \leq K$, where K denotes the number of DWT coefficients.

LSP$=\varnothing$ and **LIP**$=H$, where H is a set of all roots coordinates in the top-most lowpass subband.

Add all elements $i \in H$ with $D(i) \neq \varnothing$ to **LIS** as type-A entries.

(1) **Sorting pass**
(2) For each $i \in$ **LIP**
(3) Output $S_k(x_i)$;
(4) If $S_k(x_i) = 1$, then
move i to **LSP** and output the sign of x_i;
(5) For each $i \in$ **LIS**
(6) If i is of type-A then
(7) Output $S_k(D(i))$;
(8) If $S_k(D(i)) = 1$ then
(9) For each $j \in O(i)$

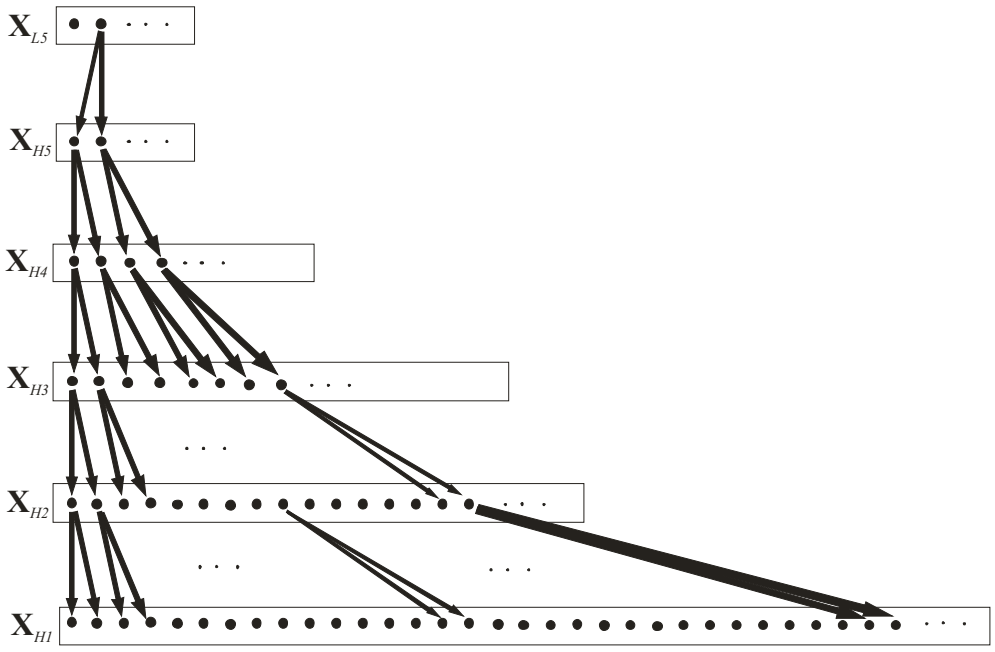

Fig. 1- A 5-level 1D-DWT spatial orientation tree of the SPIHT.

(10) Output $S_k(x_j)$;
(11) If $S_k(x_j) = 1$ then
Add *j* to **LSP** and output the sign of x_j;
(12) else append *j* to LIP;
(13) if $L(i) \neq \varnothing$ then
(14) move *i* to the end of **LIS** as an entry of type-B and go to step (5);
(15) else remove *i* from **LIS**
(16) if *i* is of type-B then
(17) output $S_k(L(i))$;
(18) if $S_k(L(i)) = 1$ then
(19) append each $j \in O(i)$ to **LIS** as an entry of type-A and remove *i* from **LIS**;

(20) **Refinement pass**
For each $i \in$ **LSP** except those included in the sorting pass, output the *k*th bit of $|x_i|$;
(21) If $k = 0$ then end; else $k = k - 1$ and go to step (1).

In the 1D-SPIHT, wavelet coefficients are arranged in a parent-offspring orientation tree in order to exploit the spatial self-similarity property of wavelet coefficients across subbands. The property implies that if a node coefficient is insignificant with respect to a given threshold, probably all nodes descending from that are insignificant too.

V. Results And Discussion

We applied nine multiwavelets on a set of 24 records from MIT-BIH database (see sec. III-A). The results are displayed in Table I, presenting the average RPD, D and CC calculated for each multiwavelet for the 24 different records. As observed from these results, cardbal2 by the means of Id prefiltering method exhibits the best results comparing the others.

In order to investigate the effect of compressing ECG signals using the cardbal2 by the means of Id prefilter (keeping N largest coefficients and discarding others, then reconstructing the signal) from the clinical point of view, three waveforms including original and reconstructed waveforms and difference between original and reconstructed signal (error) of records 107, 119 and 219 are shown in Figs. 3, 4 and 5, respectively. Note that reconstructed ECG signals are smoothed versions of the original signals, but error increases when the original signal changes abruptly.

As mentioned above, the cardbal2 multiwavelet has best performance in ECG compression, so we use it for taking multiwavelet transform from ECG signal and then apply SPIHT coding algorithm on the transform coefficients.

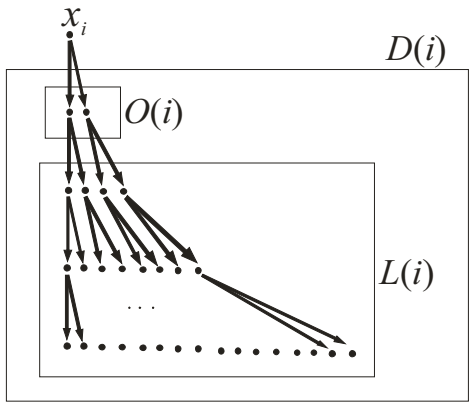

Fig. 2 - The definition of parent-offspring relationship in 1-D SPIHT

TABLE I. SUMMARY OF COMPRESSION RESULTS. PRD AND CC ARE IN PERCENT

Muliwavelet	PRD	CC	D	RMSE	SNR (dB)
GHM [1]	10.199464	98.988928	1.2055274e-002	0.043585007	17.6
GHM [2]	10.442661	98.922792	1.2616885e-002	0.044773102	17.9
CL [1]	9.3516668	99.145806	1.0522501e-002	0.040056692	18.9
CL [2]	9.1507510	99.192476	9.9911897e-003	0.039211513	18.7
SA4 [1]	9.6601638	99.102202	1.0957484e-002	0.041052841	18.9
SA4 [2]	9.2169610	99.166599	1.0220815e-002	0.038997280	18.4
bih52s [1]	11.212887	98.710719	1.6118575e-002	0.047393910	17.0
bih52s [2]	11.682872	98.589613	1.7440181e-002	0.049827301	17.3
bih54n [1]	97.075842	99.053456	1.2457205e-002	0.041163568	18.8
bih54n [2]	10.555499	98.884445	1.4600110e-002	0.044330958	18.7
bighm2 [1]	34.906194	91.819611	1.4646049e-001	0.14521119	6.1
bighm2 [2]	48.399680	85.512920	2.7072476e-001	0.19750880	8.4
Cardbal4 [3]	8.9235645	99.224869	9.5386841e-003	0.038196125	19.1
Cardbal3 [3]	9.0019501	99.208416	9.7816903e-003	0.038472232	19.0
Cardbal2 [3]	**8.8986281**	**99.230226**	**9.4111670e-003**	**0.038030973**	**19.1**

1 prefiltered with rr 2 prefiltered with app 3 prefiltered with Id

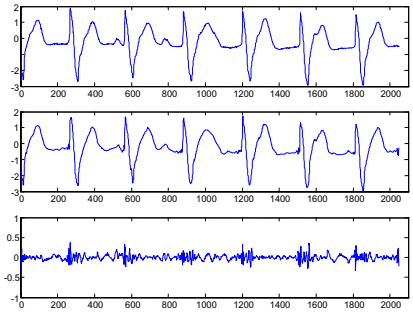

Fig. 3 – Compressing ECG using the cardbal2 with Id prefiltering method. The above figure shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 107 are used. CR=16.384, PRD=7.8842%, CC=99.6612%, D=0.006216.

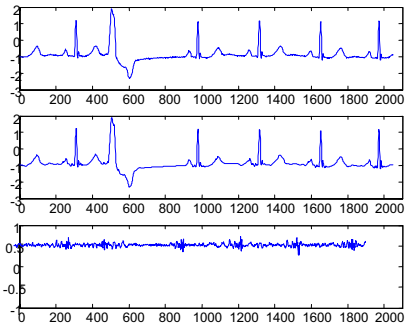

Fig. 4 – Compressing ECG using the cardbal2 with Id prefiltering method. The above figure shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 119 are used. CR=16.384, PRD=4.0136%, CC=99.6335%, D=0.0016109, RMSE=0.039487, SNR=21.3 (dB)RMSE=0.069803, SNR=21.6 (dB)

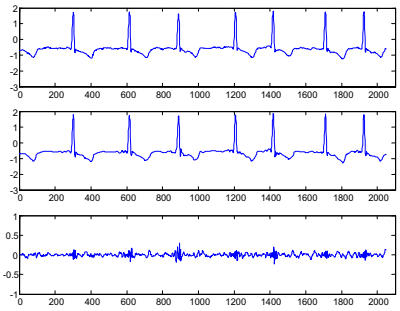

Fig. 5 – Compressing ECG using the cardbal2 with Id prefiltering method. The above figure shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 219 are used. CR=16.384, PRD=5.774%, CC=99.5286%, D=0.0033339, RMSE=0.043508, SNR=20.2 (dB)

We do similar multiwavelet decomposition, explained in sec. III-A, and then apply SPIHT on the coefficients. The stop criterion for encoder is controlled by a threshold value in the program, that allows obtaining the desired CR. By decoding the bit stream code generated by encoder, we reach to multiwavelet coefficients of signal and by taking inverse multiwavelet transform, we obtain reconstructed signal. For computing CR, we divide the original file size by the length of SPIHT coded bit stream.

The results of applying the cardbal2 multiwavelet with SPIHT coding algorithm to records 117, 119 and 100 are shown in Figs. 6 to 14. In each figure, the original and reconstructed signals and difference between them (error) is plotted. The values of CR and PRD is also shown in figures. Figs. 6-8 show the result of ECG compression for record 117 with three different CRs. Figs 9-11 show the results for record 119 with three different CRs and Figs. 12-14 show the results for record 100. From these figures we see the higher CR obtained by applying SPIHT codec. For example in Fig. 4 we have

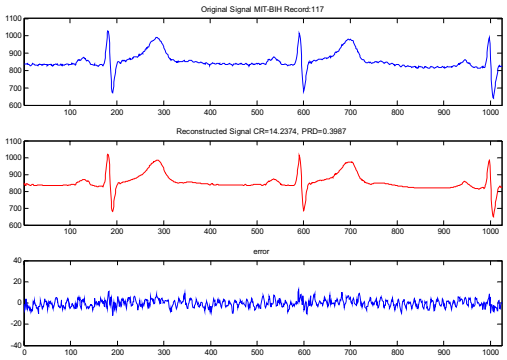

Fig 6 - Compressing ECG using the cardbal2 with Id prefiltering method. The above plot shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 117 are used. CR=14.2374, PRD=0.3987%.

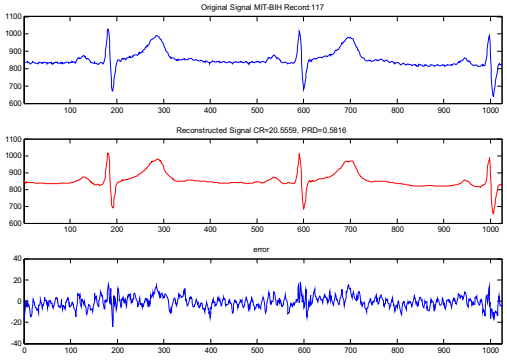

Fig. 7 – Compressing ECG using the cardbal2 with Id prefiltering method. The above plot shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 117 are used. CR=20.5559, PRD=0.5816%.

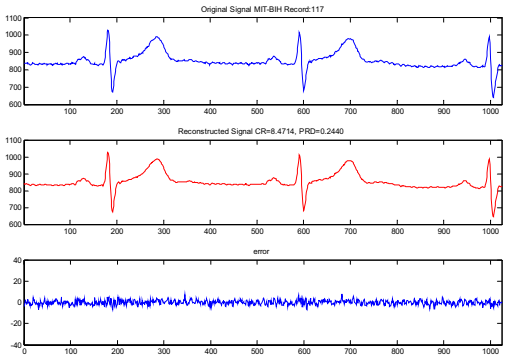

Fig. 8 – Compressing ECG using the cardbal2 with Id prefiltering method. The above plot shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 117 are used. CR=8.4714, PRD=0.2440%.

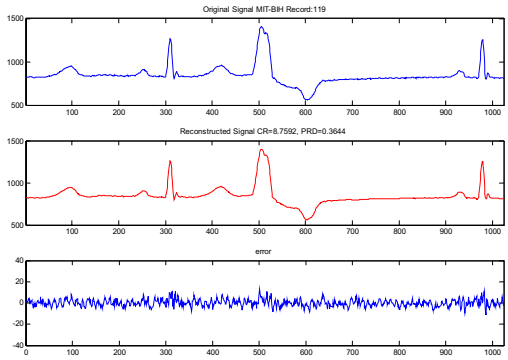

Fig. 9 – Compressing ECG using the cardbal2 with Id prefiltering method. The above plot shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 119 are used. CR=8.7592, PRD=0.3644%.

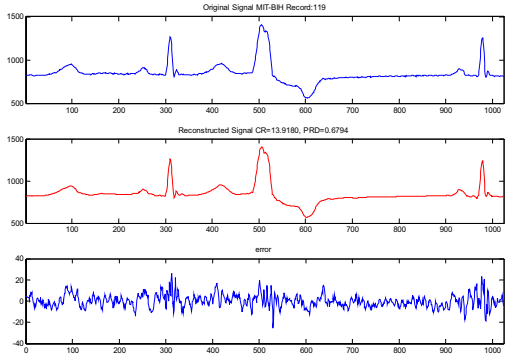

Fig. 10 – Compressing ECG using the cardbal2 with Id prefiltering method. The above plot shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 119 are used. CR=13.9180, PRD=0.6794%.

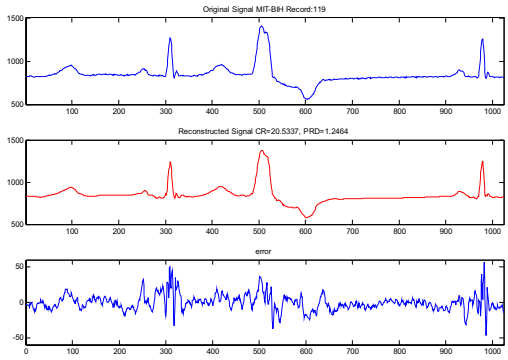

Fig. 11 – Compressing ECG using the cardbal2 with Id prefiltering method. The above plot shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 119 are used. CR=20.5337, PRD=0.1.2464%.

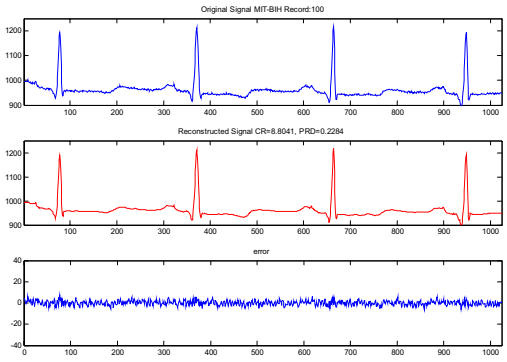

Fig. 12 – Compressing ECG using the cardbal2 with Id prefiltering method. The above plot shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 100 are used. CR=8.8041, PRD=0.2284%.

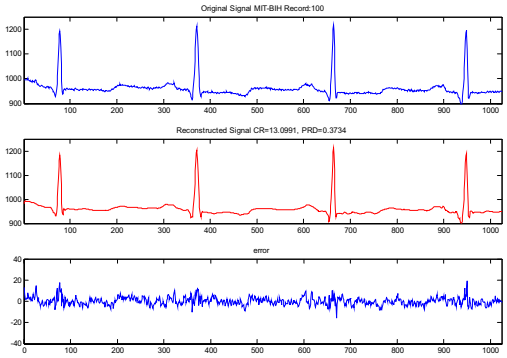

Fig. 13 – Compressing ECG using the cardbal2 with Id prefiltering method. The above plot shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 100 are used. CR=13.0991, PRD=0.3734%.

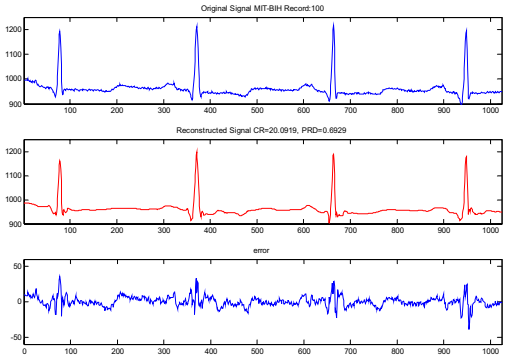

Fig. 14 – Compressing ECG using the cardbal2 with Id prefiltering method. The above plot shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 100 are used. CR=20.0919, PRD=0.6929%.

reconstructed record 119 with CR=16.384 and PRD=4.01%, but in Fig. 10, with a close CR to this value (CR=13.918) we have a much lower PRD, which is 0.6794%. Other results show that just by increasing CR to very high values we obtain a PRD near previous values (between 5% to 10%).

VI. Conclusion

In this paper, we studied the optimum multiwavelet for compressing the ECG signal with respect to some assessment criteria and found that cardbal2 has the best performance. Then we applied it with SPIHT codec to three records from MIT-BIH database and investigate the results. It should be noted that a further improvement in results may be achieved with other multiwavelet bases and new prefiltering approaches. In this paper we only used inter-beat dependencies. For future work we are going to construct 2D-ECG array from one dimensional ECG toward using intra-beat and inter-beat dependencies to achieve maximum data compression.

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