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Testing the Bayesian model of perceived speed

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Abstract

In a recent Bayesian model by Weiss, Simoncelli, and Adelson, motion perception is biased by a prior favoring slow speeds. This model predicts qualitatively an impressive variety of phenomena, including the dependence of perceived speed on contrast. We show that the model can also generate quantitative predictions: for a drifting grating with contrast c, perceived speed is proportional to $c^q/(k^q + c^q)$, with k, q constants. We tested this expression on measurements of perceived speed as a function of contrast. Observers indicated the slower of two drifting gratings, a *test* and a *standard*. For each test contrast we found the test speed that appeared to match the standard speed. The model fits the data, but only if q is less than 2, the value it would have if the internal representation of contrast were linear. The Bayesian model can make correct quantitative predictions, but needs to be extended to incorporate a more realistic, nonlinear representation of contrast.

Keywords: Motion perception; Speed sensitivity; Illusion; Bayesian model; Contrast

1. Introduction

Work in the last decade has led to a model of motion perception that is simple and powerful (Ascher & Grzywacz, 2000; Simoncelli, Adelson, & Heeger, 1991; Weiss & Adelson, 1998; Weiss, Simoncelli, & Adelson, 2002). According to this model, observers are Bayesian: perception of motion results from the product of a *likelihood* based on the evidence at hand and a *prior* founded on a priori knowledge (Fig. 1A). The product is called the *posterior*:

$$\text{Posterior} = \text{Prior} \times \text{Likelihood}.$$

Perceived velocity $v_{\text{perceived}}$ is the one that maximizes the posterior:

$$\text{Posterior}(v_{\text{perceived}}) = \max(\text{Posterior}).$$

The likelihood is a probability distribution centered on the physical speed of the stimulus, v_{real}. The spread of this distribution is due to noise in the measurement of speed. The prior is a probability distribution centered on a speed of zero, so that in the absence of further information (e.g. in the dark) one does not assume motion in any particular direction. The peak of the posterior, perceived speed $v_{\text{perceived}}$, is lower than the physical stimulus speed v_{real} because the prior biases the estimate towards zero.

This model explains qualitatively an impressive variety of motion perception phenomena (Weiss & Adelson, 1998; Weiss et al., 2002), including the classic observation that stimuli appear to move slower at low contrast than at high contrast (Blakemore & Snowden, 1999; Stone & Thompson, 1992; Thompson, 1982). This effect can be strong enough to affect our everyday life. For example, it might underlie a tendency for drivers to accelerate when visibility is reduced by fog (Snowden, Stimpson, & Ruddle, 1998).

The model predicts the dependence of perceived speed on contrast because decreasing contrast increases the noise in the measurement of v_{real}, hence broadening the likelihood and allowing the prior to exert more weight. At low contrast, therefore, $v_{\text{perceived}}$ is close to zero (Fig. 1A). At high contrast, instead, noise is low, so likelihood is sharp (Fig. 1C); therefore, posterior is similar to likelihood, and $v_{\text{perceived}}$ is close to v_{real}.

We asked whether the model can predict the contrast dependence of perceived speed quantitatively. We expressed its prediction in a closed form equation, which holds when stimuli are drifting gratings. We then fitted this equation to psychophysical measurements of perceived speed.

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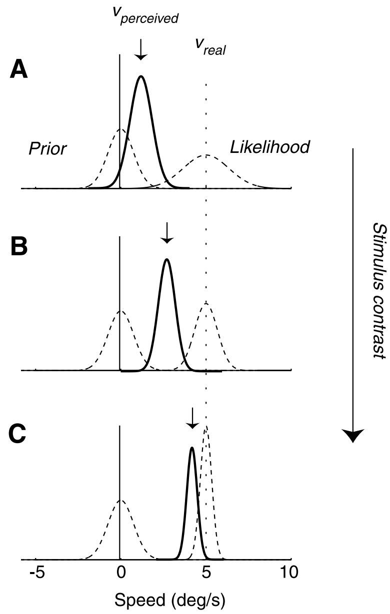

Fig. 1. Predictions of the Bayesian model of motion perception, simplified to the case of one-dimensional motion. Dashed curves indicate prior, which is centered on zero speed, and likelihood, which is centered on stimulus speed $v_{\mathrm{real}} = 5$ deg/s. Continuous curve is the posterior, which is obtained by multiplying prior and likelihood. The location of its peak (arrows) is perceived speed $v_{\mathrm{perceived}}$. Rows correspond to different stimulus contrasts. (A) Low contrast (B) intermediate contrast and (C) high contrast.

An earlier version of this work has appeared in abstract form (Hürlimann, Kiper, & Carandini, 2000).

2. Model

The Bayesian model of motion perception of Weiss, Simoncelli and Adelson is formulated on the basis of a simplifying assumption, that the function describing the likelihood of different velocities is Gaussian (Simoncelli et al., 1991). For one-dimensional motion, this Gaussian is

$$\mathrm{Likelihood}(v) = \exp[-(I_x v - I_t)^2/(2\sigma_t^2)]$$

where I_x and I_t are the derivatives of the image in space and time, and σ_t is measurement noise.

For drifting sinusoidal gratings integrated over a large enough window of space and time we can assume this likelihood to be independent of space and time. If we note that the physical speed of the stimulus is $v_{\mathrm{real}} = I_t/I_x$ we can rewrite the likelihood as a Gaussian with mean v_{real} and variance σ_t/I_x:

$$\mathrm{Likelihood} = \mathrm{Gaussian}[v_{\mathrm{real}}, \sigma_t/I_x]$$

For a drifting sinusoid, the derivative in space of the light intensity distribution, I_x, is proportional to stimulus contrast c. We can then introduce a new constant σ and write

$$\mathrm{Likelihood} = \mathrm{Gaussian}[v_{\mathrm{real}}, \sigma/c].$$

Because in the Bayesian model the prior is itself a Gaussian (Weiss & Adelson, 1998; Weiss et al., 2002)

$$\mathrm{Prior} = \mathrm{Gaussian}[0, \sigma_{\mathrm{prior}}],$$

and the maximum of Gaussian $[m_1, \sigma_1] * \mathrm{Gaussian}[m_2, \sigma_2]$ lies at $(m_1/\sigma_1^2 + m_2\sigma_2^2)/(1/\sigma_1^2 + 1/\sigma_2^2)$ we can write a simple equation for the dependence of perceived speed on the contrast of a drifting grating:

$$v_{\mathrm{perceived}} = v_{\mathrm{real}} \frac{c^2}{k^2 + c^2}, \quad (1)$$

where $k = \sigma/\sigma_{\mathrm{prior}}$ is a constant.

Eq. (1) was derived by assuming that the representation of contrast in the motion system is linear. This assumption is likely to be wrong. For example, as contrast grows responses in cortical area MT of the macaque show clear saturation (Sclar, Maunsell, & Lennie, 1990). This saturation is thought to be due to contrast gain control, which lets the system operate linearly while reducing responsiveness (Heeger, 1992; Shapley & Victor, 1978; Simoncelli & Heeger, 1998). Under this assumption, one can repeat the derivation above while modeling contrast responses as going through a compressive nonlinearity such as an exponent <1. One then obtains

$$v_{\mathrm{perceived}} = v_{\mathrm{real}} \frac{c^q}{k^q + c^q}, \quad (2)$$

which is more general than Eq. (1). A value $q < 2$ indicates a compressive nonlinear representation of contrast.

Eqs. (1) and (2) predict a sigmoidal dependence of perceived speed on contrast. As contrast grows, perceived speed tends towards real speed. The parameter k determines at which contrast perceived speed is half of real speed. The exponent q determines the steepness of the dependence of perceived speed on contrast.

To test the validity of the model, one needs to measure perceived speed. This measure can be performed indirectly, by asking a subject to compare the speed v_t of a *test* stimulus with the speed v_s of a *standard* stimulus and finding the v_t that matches v_s. According to Eq. (2), these speeds obey

$$v_t = v_s \frac{c_s^q}{k^q + c_s^q} \frac{k^q + c_t^q}{c_t^q}. \quad (3)$$

This expression has only two free parameters, k and q, and dictates how the matching test speed v_t should depend on test contrast c_t given a standard grating with speed v_s and contrast c_s.

3. Methods

To test the predictions of the model we measured matching test speed as a function of test contrast. Subjects indicated the slower of two gratings drifting in the same direction, a *standard* and a *test* (Fig. 2). We employed four standard gratings, with contrast 10% or 50% and speed 1 or 2 deg/s. The contrast of the test grating had one of nine values between 2% and 100%. Stimuli were divided in four blocks, one for each standard grating. For each of the nine conditions in each block, test speed was determined by an adaptive psychometric procedure (QUEST, Watson & Pelli, 1983), which aimed for the speed yielding 50% of responses. This procedure was given 40–50 trials to converge. We then fitted the responses with a Weibull psychometric function (using the maximum likelihood method, Watson, 1979). The result of each block was a set of nine test speeds that appeared to match the standard speed.

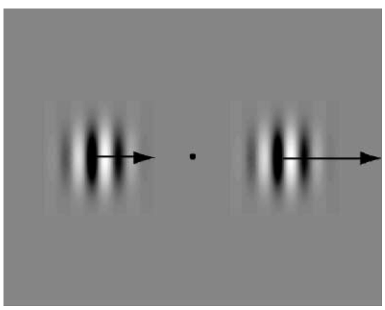

Fig. 2. Stimuli. Sinusoidal gratings (1.5 cycles/deg) were enclosed in Gaussian windows (diameter 3 deg, duration 1 s) to the left and to the right of a fixation mark (eccentricity 4.5 deg, binocular viewing, distance 170 cm). Arrows indicate direction of motion and speed.

Stimuli were generated with the Psychophysics Toolbox (Brainard, 1997; Pelli, 1997) and presented on a calibrated 21″ CRT (Sony Multiscan G500, mean luminance 37 cd/m^2) driven by a graphics board with 159 Hz refresh rate (VillageTronic MacPicasso 850). To minimize the effects of light adaptation, mean luminance was kept uniform in space and time throughout the experiment (with the exception of the fixation mark). To minimize the effects of contrast adaptation, we randomized the spatial position and direction of standard and test, as well as test contrast. Subjects were the three authors.

4. Results

The results of our experiment (Fig. 3) are consistent with similar measurements in the literature (e.g. Dougherty, Press, & Wandell, 1999; Hawken, Gegenfurtner, & Tang, 1994; Muller & Greenlee, 1994; Stone & Thompson, 1992; Thompson, 1982). When test and standard gratings have the same contrast, they are physically identical, so matching test speed has to be close (within measurement error) to standard speed.

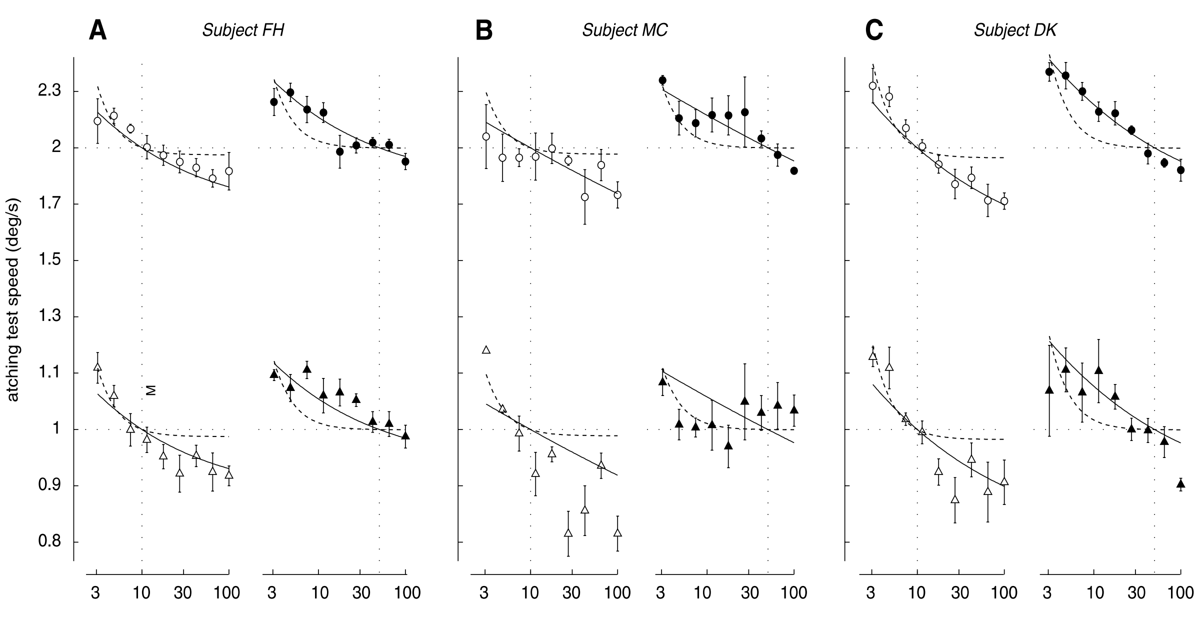

Fig. 3. Dependence of matching speed on contrast, fitted with model predictions. Panels contain data from one observer and four standard gratings, with contrast $c_s = 10\%$ (open symbols) and 50% (closed symbols) and speed $v_s = 1$ deg/s (triangles) and 2 deg/s (circles). Vertical dotted lines indicate c_s. Horizontal dotted lines indicate v_s. Error bars are two standard errors of the mean. Scaling on all axes is logarithmic. Continuous curves are fits of the model in Eq. (3). Dashed curves are fits obtained by fixing the exponent to $q = 2$. (A) subject FH, $k = 0.1\%$, $q = 0.4$, (B) subject MC, $k = 20.8\%$, $q = 0.1$ and (C) Subject DK, $k = 0.3\%$, $q = 0.3$.

When test contrast is less than standard contrast, the test has to move faster to appear to be moving at the same speed as the standard. At low standard contrast (*open symbols*) it is possible to increase test contrast so that the test appears to move faster than the standard (matched test speed is below standard speed). This is observed more rarely at high standard contrast (*closed symbols*).

We fitted these measurements with the predictions of Eq. (3), allowing the two parameters to vary across observers, but holding them fixed across conditions.

While the fits are not perfect, the model does capture the main features of the data (Fig. 3, continuous curves). The model explains 85% of the variance in the data for subject FH (Fig. 3A) and 83% of the variance in the data for subject DK (Fig. 3C). The worst performance of the model is with subject MC, where the model explains only 54% of the variance in the data (Fig. 3B). This subject was, however, the least reliable: there is high variance in the data themselves. Overall, considering that we are fitting 36 data points from four conditions with only two free parameters, we take these results to be encouraging.

The model captures the dependence of perceived speed on contrast only if one allows for a nonlinear internal representation of contrast. The exponent q estimated by the model varies from 0.1 (for MC) to 0.4 (for FH), substantially lower than the value of 2 that is predicted if the internal representation of contrast were linear. Indeed, if we constrain the exponent to $q = 2$ (as in Eq. (1)) we get unsatisfactory fits (Fig. 3, dashed curves). The percentage of variance explained by the model drops to 52%, 36% and 54%. These values are low. Indeed, the fits show excessive saturation and predict that the standard should appear faster than the test for most test contrasts.

The values of the exponent q predicted by the fits seem qualitatively appropriate given that our contrasts are well above detection threshold. Indeed, while near threshold one would expect the nonlinearity to be expansive, above threshold the nonlinearity is widely thought to be compressive (e.g. Boynton, Demb, Glover, & Heeger, 1999).

5. Discussion

We have shown that the Bayesian model of motion perception makes quantitative predictions of the dependence of perceived speed on contrast of drifting gratings. When a more realistic representation of contrast is introduced these predictions are in broad agreement with the data.

A sigmoidal dependence of perceived speed on contrast has been described in other studies, which have employed expressions similar to our Eq. (2) to describe the data (e.g. Dougherty et al., 1999; Muller & Greenlee, 1994). Our derivation of the predictions of the Bayesian model provides the missing theoretical foundation for such expressions.

On the other hand, a number of other studies have demonstrated a dependence of perceived speed on contrast that is largely linear (e.g. Hawken et al., 1994; Stone & Thompson, 1992; Thompson, 1982). Even our own data would not be fitted too badly by lines. A line differs most from a sigmoid at low contrasts and at high contrasts. We were not able to test perceived speed at low contrasts, because below about 3% contrast stimulus detection was impaired, and subjects were not even able to match the speeds of two identical gratings. We were however able to measure the effects at high contrasts, and in our data there is evidence for saturation (Fig. 3).

A possible explanation for the discrepancies in the literature might lie in the spatiotemporal configuration of the stimuli. It is possible that for some stimuli one can measure perceived speed at very low contrasts, and find a sigmoidal relationship between perceived speed and contrast. For other stimuli one can only test the regime where this relationship appears linear.

Nevertheless, as it stands the Bayesian model is unlikely to explain the entire variety of effects described in the literature. The reduction of perceived speed with decreasing contrast varies from observer to observer and depends on a number of visual attributes; at high speeds it might be diminished or even reversed (Blakemore & Snowden, 1999). Outside the restricted range of speeds that we tested, the model might have been less successful.

To summarize, the Bayesian model is not only able to predict a broad range of motion perception phenomena qualitatively (Weiss & Adelson, 1998; Weiss et al., 2002), but can also be made quantitative and is able to predict actual data. We have shown, however, that the model can predict the effects of contrast on perceived speed much better if it is extended to incorporate a nonlinear dependence of response on contrast. In the encounter with data, such an elegant model has to lose some of its simplicity.

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