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Periodic economic cycles: the effect of evolution towards criticality, and control

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Abstract. When a system becomes unstable or noise becomes excessive, often regulations in the form of limiters (barriers obstructing excursions in undesired directions) are imposed. It is hoped that under the influence of this element, the system can be calmed and its behaviour optimized. We consider a simple noisy nonlinear economics model that self-organizes towards criticality. We demonstrate that the inherent effect of limiters is the emergence of stable cycles, and that the limiters need to be implemented with care in order to obtain an optimized system response. In particular, implementing the limiter at maximal system response is generally a suboptimal solution. We find that the system average is generally optimized by controlling a period-one cycle. Furthermore, we provide optimality conditions for the case where the control is restricted to being on the natural system behaviour.

Strong interventions are needed in order to acquire the period-one cycle. In democratic countries, a transparent control policy would be a necessary condition for its implementation. The framework discussed provides such a model.

Keywords: critical phenomena of socio-economic systems, financial instruments and regulation, nonlinear dynamics

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Contents

1. Introduction

Economic booms and bouts affect modern societies strongly, with a direct impact on individuals' biographies. In western economies, cycles have been a ubiquitous (undesired) observation. Among the most remarkable, Kitchin cycles emerged [1]. Until the 1970s, as the legacy of Keynes [2], cycles were regarded as primarily due to variations in demand (company investments and household consumption). As a consequence, economic analysis focused on monetary and fiscal measures to offset demand shocks. During the 1970s, it became obvious that stabilization policies based on this theory failed. Shocks on the supply side, in the form of rising oil prices and declining productivity growth, emerged as equally crucial for the generation of cycles. In a paper published in 1982, Kydland and Prescott [3] offered new approaches to the control of macroeconomic developments. One of their conclusions was that the control should be kept constant throughout a cycle, in order to minimize negative effects.

Cycles and crises may be inherent to the principles on which our economics is based. However, if they could be predicted and their origin understood, they might be engineered to take a softer course. An extreme form of this approach was taken in the centrally planned economies in the former socialist countries. In order to deal with this problem in democratic societies, it is necessary to be able to communicate a sufficiently simple optimality policy. For obtaining it, the understanding of the response to control in simple economical models may provide important guidelines [4]. The prediction problem of economics is closely related to the one in chaotic processes, where strategies for overcoming it have been developed. Although the question of to what extent real economies can be classified as chaotic can readily be disputed, low-dimensional chaotic models yield insight into the mechanisms that govern the response of economics to control policies. Interestingly, in early implementations of the optimal control program, it was already found that control mechanisms themselves may induce chaotic behaviour [5]–[7] and render optimal control impossible. As a general mechanism inherent in many of these examples, chaos is induced by a preference function that depends on past experience.

This delay mechanism naturally makes a dynamical system infinite dimensional, which has the tendency of resulting in a chaotic behaviour.

Chaos is composed of an infinite number of unstable periodic cycles of increasing periodicities. In order to exploit this reservoir of characteristic system behaviours, elaborate methods have been developed to stabilize (or 'control') intrinsically unstable orbits, using only small control signals [9]–[12]. By the more detailed study of the potential of these control methods in economics [13]–[17], several limiting factors were identified. As a first shortcoming, the inherent latency of most of the above control approaches emerged. In the context of quickly changing economics, control, however, is required to be fast. As a second shortcoming, most economic data cannot be collected continuously. This renders the application of the standard control methods, that are based on the explicit knowledge of the geometry of the economical dynamics, tedious, and targeting methods, designed to improve the speed of convergence towards the desired solutions, inefficient. Moreover, the large amount of strong noise that is characteristic for economics tends to veil these structures. As a third shortcoming, the control should be realizable as a simple economics policy. For control strategies that are based on past observations (e.g. statistical data from the preceding year), this is not easily achievable. Moreover, these control strategies lead to policy functions involving time delays [4], which often entrain chaotic behaviour, as in the above-mentioned pioneering examples [5]–[7]. These observations apply in particular to time-delayed feedback control methods [17, 18] that for some time were proposed as a means of controlling financial markets. Due to these problems, the interest in the application of dynamical systems methods for the control of economic dynamics has decreased subsequently.

In our contribution, we identify a general principle that naturally generates cycles in economical models. We then demonstrate a detailed mechanism of how cycles are additionally introduced when applying even the simplest control strategies. The insights obtained add a new facet to the control advice of Kydland and Prescott: the optimal system behaviour is not obtained by controlling the natural cycle, but is achieved by a controlled period-one orbit. This does not only require a control policy that is kept fixed through time. To acquire the period-one state, a strong initial control effort is generally required, and control must be permanently maintained. In order to control the system on a period-one orbit, it may be advantageous if the system is in the chaotic regime. We show how the regime can be identified from the system's response to control.

The structure of this paper is as follows. First, the logistic map is introduced as a simple, but generic, model of economics. Using this model, a primary source of cycles is identified. Then the principles of the simplest natural control method are explained, and the laws underlying the generation of (super)stable cycles by means of the control are outlined. Finally, the control method is applied to noisy stable and chaotic time series of the logistic map, and the conditions for optimal control are determined.

2. A simple view on economics

When exponential growth is possible, real economies have little problem. It is mostly when the limits of the economic systems are reached that their prediction becomes difficult. From the mathematical point of view, this is due to the nonlinearities that are required to keep the system within the boundaries. Economies naturally tend towards the recruitment

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of all available resources. This drives them towards the boundaries and fosters a natural tendency of the system to evolve towards maximally developed nonlinearities. We can thus describe economics in a simplified and abstract way in terms of a parameter indicating the degree of globalization of resources (nonlinearity parameter a), and a dynamical parameter x expressing a generalized consumption. The evolution of this simple model of economics takes place on three timescales: a slow one which modifies parameter a, an intermediate-term variable x that is assumed to be deterministic, and momentary perturbations that are included in x in the form of noise. The underlying deterministic system is defined by the property that for states far from full exploitation of the resources, the consumption can grow almost linearly. Close to maximal exploitation, the next consumption is required to be small, to let the system recover. Over a large parameter range of small a (local economics), this behaviour, however, is avoided and a state of quasi-constant consumption emerges.

A most simple and generic setting for modelling this dynamics is provided by the iterated logistic map

$$f: \quad [0,1] \to [0,1]: \quad x_{n+1} = ax_n(1-x_n). \tag{1}$$

The above-mentioned self-organization towards an ever-growing exploitation of the phase space $[0,1]$ is reflected in a slow increase of the order parameter a towards $a=4$. At $a=4$, it can easily be seen how the nonlinearity keeps the 'orbits' x_n away from the boundary: starting with small values, x_n increases almost linearly (with factor a). As soon as x_n approaches the upper phase-space boundary (at $x_n = a/4 = 1$), this is counterbalanced by the factor $1-x_n$. If a is increased further, large-scale erratic behaviour sets in, as the process is no longer confined to the previously invariant unit interval. After a potentially chaotic transient, the system settles into a new area of stability, where the same scenario takes place anew, starting at rescaled small a. We believe that in particular the effects of technical shocks may be adequately described in this framework.

On its way towards the globalization of resources ($a \to 4$), the system undergoes a continued period-doubling bifurcation route, where a stable period-one solution is transformed, over a cascade of stable orbits of increasing orders 2^n (where $n = 2,3,4\ldots$), into a chaotic solution (the Feigenbaum period-doubling cascade [8]). Using renormalization theory, it can be shown that in order to reach the next bifurcation, a progresses geometrically, with factor $q \approx 1/4.67$. This implies that the transition point to period infinity is reached within a finite interval of a. Beyond this period-doubling accumulation point, chaos is possible and abundant [19, 20]. The properties exhibited by the logistic map are characteristic for a large universality class of unimodal maps (of which it is the simplest representative; see e.g. [21]). Our model is thus characteristic for the whole class of systems that are subject to such a process of self-organization.

Equation (1) has previously been used in a number of nonlinear models of economics. In an early example given by Benhabib and Day [7], under suitable conditions economics was found to be described by the logistic map. This was an early indication that economics could eventually become chaotic. In their model, there is a competition between the demand for two goods. The preference for one good is a function of past experience (this is taken account of by an iterative implementation) and of a constraint formulated in terms of a fixed budget. The nonlinearity parameter a is a decreasing function of the prices. Other simple examples that naturally lead to the logistic map can easily be constructed

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along the following lines. Consider, e.g., whaling in the Northern Atlantic Ocean. If the whaling fleet is small (captured by $a \ll 1$), the annual catch x_n will be small and affect the whale population little, so x_n will stay at a quasi-fixed point. An increase of a will raise the average catch $\bar{x}$. Larger ships will start venturing to the whole of the Atlantic Ocean. At the point when we start to exploit a considerable part of the whole system ($a \to 4$), the fixed-point behaviour naturally ceases to hold. After a situation of almost complete exploitation ($x_n \approx a/4$), the system needs an extended time to recover. Novel technologies may annihilate the constraints that originally defined the confinement to the unit interval. The universality underlying the above-discussed route to ever more complex dynamical behaviour, however, implies that under the new constraints, the whole process will repeat, leading to a cascade of such processes, theoretically *ad infinitum*.

3. Effects of simple control

Whereas the usage of the logistic map as a simple, yet generic, model of macroscopic economics seems reasonably motivated, in real economics the demand x is characterized by strong short-term fluctuations, often of local or external origin. Whereas in the case of small a such perturbations are stabilized by the system itself, for larger a they lead to ever more long-lived erratic excursions. To incorporate these fluctuations within our model, we perturb x with multiplicative noise, for simplicity chosen uniformly distributed over a finite interval. The size of the noise sampling interval, in the following denoted by str, is a measure for the amount of noise. To render economics predictable under these circumstances, it is natural to apply a control mechanism to x. For this, a sufficiently simple control tool is needed, whose properties are well understood and which does not additionally complicate the behaviour of the system. As a natural candidate, control by means of placing a limiting value on x that the system is not allowed to cross, can be—and in reality often is—imposed. In figure 1, three time series generated from this model are displayed. For the first series, the system was tuned so as to generate a noisy superstable period-four orbit (for the definition of (super)stability of orbits see e.g. [21]). For the second series, a limiter at the highest cycle point was inserted, whereas in the third series the control was on the unstable period-one orbit. It is easily seen that the period-one orbit yields the highest average value.

Recently, exact results for this so-called hard limiter control (HLC) have been obtained. For reasons of convenience, we will expose their nontrivial essence. By introducing a limiter, orbits that sojourn in the forbidden area are eliminated (see figure 2). Modified in this way, the system tends to replace previously chaotic with periodic behaviour. By gradually restricting the phase space, it is possible to transfer initially chaotic into ever simpler periodic motion. When the modified system is tuned in such a way that the control mechanism is only marginally effective, the controlled orbit runs in the close neighbourhood of an orbit of the uncontrolled system. In a series of papers [22]–[26], this control approach was successfully applied in different experimental settings, and its properties were fully analysed.

Flat-topped maps are the proper paradigm for studying HLC [25, 26]. They are obtained by replacing the peak region of a map by a horizontal line at height h, which limits the phase space to $\{x \mid x < h\}$. A detailed analysis shows that the class of flat-topped maps shares a number of remarkable topological and metric features [25].

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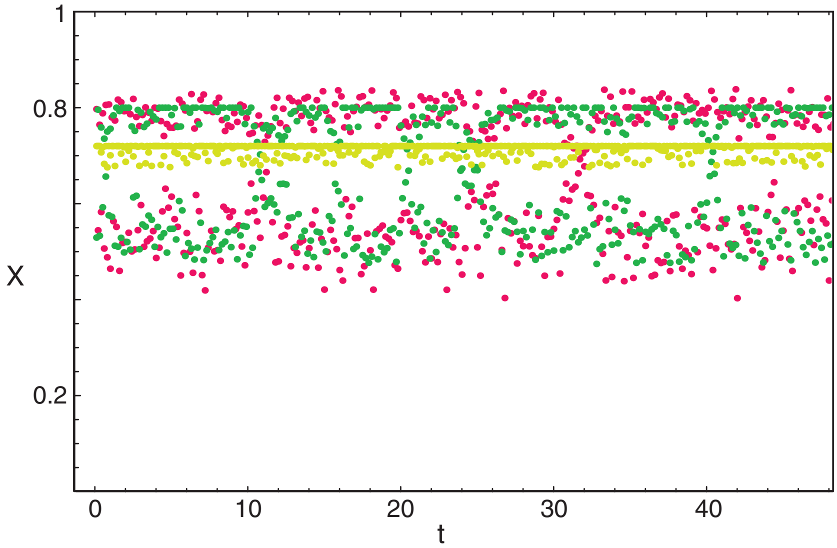

Figure 1. Noisy time series of a superstable period-four orbit (str = 0.02), uncontrolled (red), controlled in the maximal cycle point (dark green), and controlled in the unstable period-one orbit (light green). The period-one orbit yields the highest average.

Figure 3(a) shows the generic bifurcation diagram exposed by this class, as a function of the natural control parameter h (for optimized display, the diagram of the flat-topped tent map is shown). It is observed that the controlled map undergoes a period-doubling bifurcation cascade, leading to long, seemingly chaotic, orbits. However, in this system, no chaotic orbits are allowed. By ergodicity, each orbit will eventually pass by the control segment, from where on the orbit is periodic, as landing on the control segment entrains a zero slope. (A multiplication of a product of slopes, as involved in the calculation of Lyapunov exponents, by a zero factor, leads to a zero product. This immediately sets the Lyapunov exponent to minus infinity, which implies superstable periodic behaviour.)

Period-doubling cascades are characterized by two constants, α and δ [8]. The constant α describes the asymptotic scaling of the fork openings of subsequent period doublings, whereas δ represents the scaling of the intervals of period 2^n to that of period 2^{n-1} near the period-doubling accumulation point, i.e. at the transition to chaos. The observed period-doubling bifurcation cascades are typical for flat-topped maps (or the control method) and differ in scaling from the Feigenbaum case. The ratio of the bifurcation fork openings within forks of the same periodicity now depends on the derivative of the map, and is therefore non-universal.

The scalings induced by HLC also explain the large-scale repetitive star-like bifurcation structures and the adjacent repetitive empty bands (whose positions are indicated in figure 3(b) by the large and the small circles, respectively). It is easy to see that the asymptotic scalings of these repetitive structure stars are both given by the derivative of the leftmost fixed point of the map. As a consequence, both scalings are again non-universal. In applications, the time required to arrive in a close neighbourhood of the target orbit is an important characteristic of the control method. With the classical methods, unstable periodic orbits can only be controlled when the system is already in

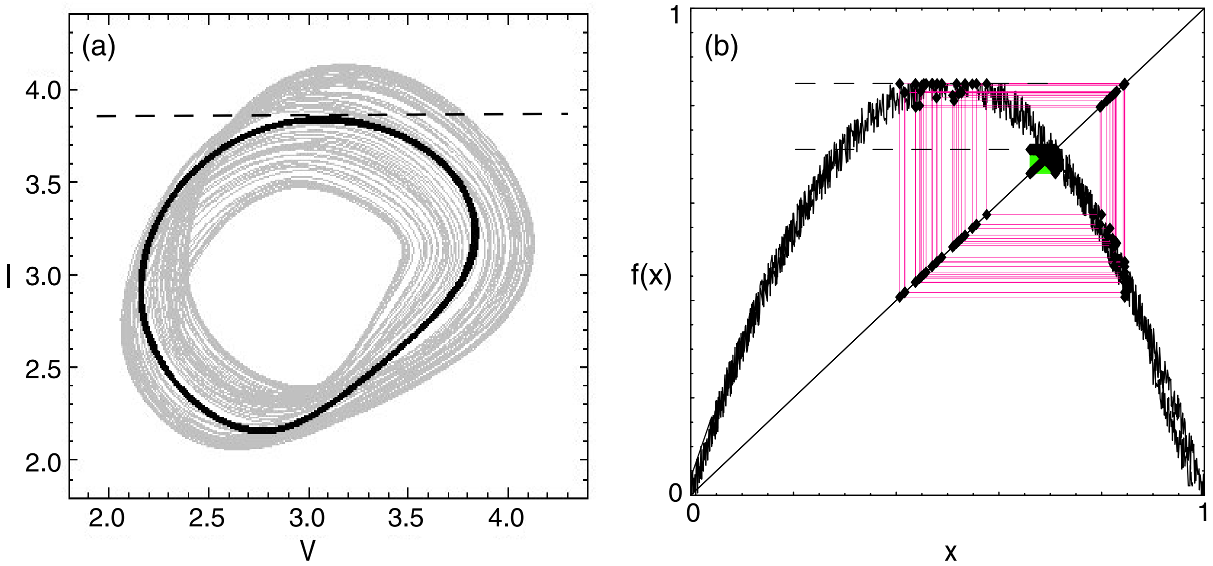

Figure 2. HLC for time-continuous and discrete dynamical systems. Limiter positions are indicated by dashed lines. (a) HLC changes chaotic into period-one behaviour (modified from Corron *et al* [24]). (b) HLC for the noisy logistic map. Placement of the limiter around the maximum of the map preserves the natural noisy period-two orbit (red). For lower placement, a modified period-one behaviour is obtained (green).

the vicinity of the target orbit. As the initial transients can become very long, algorithms have been designed to speed up this process [27, 28]. HLC renders targeting algorithms obsolete, as the control-time problem is equivalent to a strange repeller-escape (control is achieved as soon as the orbit lands on the flat top). As a consequence, the convergence onto the selected orbit is exponential [25].

These properties of 1D HLC systems fully describe the effects generated by the limiter control. Due to the control, only periodic behaviour is possible. Period-doubling cascades that have a superexponential scaling $\delta^{-1}(n) \sim 2^{-2^n}$ [25], and therefore are not of the Feigenbaum type, emerge in the control space. The convergence onto controlled orbits is exponential. Controlled orbits are unmodified original orbits only at bifurcation points of the controlled map. For generic one-parameter families of maps, all bifurcation points are regular, and isolated in a compact space. As a consequence, their Lebesgue measure is zero. These properties substantially modify the uncontrolled system behaviour.

4. Natural versus control-induced cycles

It is a widespread misunderstanding that control methods only apply to inherently unstable systems. Unmodified control methods can be used to control unstable orbits of inherently stable systems. In either case, the control should be only minimally active. In the noise-free case, the control is optimal, if after an initial phase the controller no longer experiences any noticeable strain. This is the case at the bifurcation points of the controlled map. Questions that remain are whether a corresponding statement also holds true for noisy systems, and which of the orbits should be controlled.

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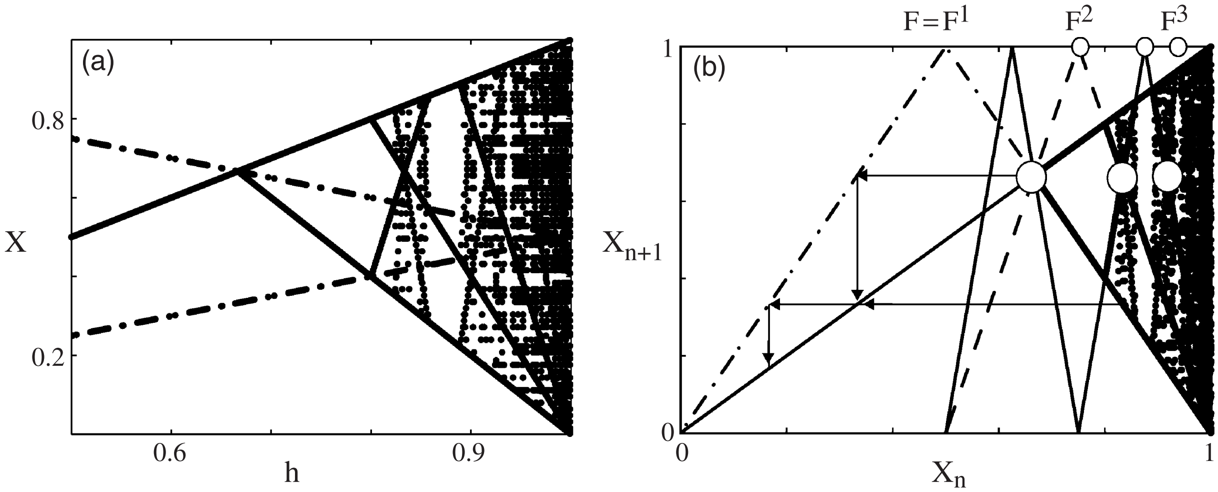

Figure 3. (a) Generic bifurcation diagram of flat-topped maps. The (for display reasons: tent) map is drawn over the vertical axis x (broken lines). To obtain the controlled map at control parameter h, replace the (rightwards pointing) peak of the map by a vertical segment positioned at h. The asymptotic controlled orbit points are also displayed with abscissa h, giving rise to a bifurcation diagram. (b) Relation between the n-fold iterates of F (graphs F^n, $n = 1, 2, 3$, shown by dashed–dotted, dashed, and full lines) and the scaling of the 'stars' (large circles): back-iterations (arrows) of the period-one fixed point $x = 2/3$ yield successive star locations. Their scaling is therefore determined by the derivative $F'(0)$. A similar argument applies for the size of the 'windows' (whose x-values are located around the small circles).

As the economic system evolves, it will be in a noisy, but stable period-one state. This is a convenient economic behaviour. Predictions and forecasts are simple to make. To reduce the noise, the limiter will be placed around the periodic point. As the system turns into a period-two one, the question emerges of whether to maintain the unstable period-one cycle, or whether to move on to the stable period-two one. We will argue that maintaining the period-one cycle is preferable, from most economics aspects. The predictions for these systems are simpler, and lead to simpler economic policies. Many economic indicators (taxes, budgets, etc) are evaluated over a period of one year. Moreover, the period-one x-average will be generally higher than that of the controlled period-two case, as well as any other higher cycle. This appears counter-intuitive, since the natural tendency to relax back to the 'natural' system state has to be compensated for by the control. From the convexity of the nonlinear map, however, it is easy to prove that our claim holds. To change a natural higher periodic behaviour into a period-one state generally requires a relatively strong initial control action. That this is beneficial appears to be counter-intuitive again, and needs to be communicated in an accompanied economic policy statement.

When the timescale over which the external parameter a varies becomes comparable to the cycle wavelength, the optimality of the above-described control may break down, as continued adjustments need to be made in order to follow the changing period-one location. In this case, it may be preferable to control a natural cycle. The most obvious

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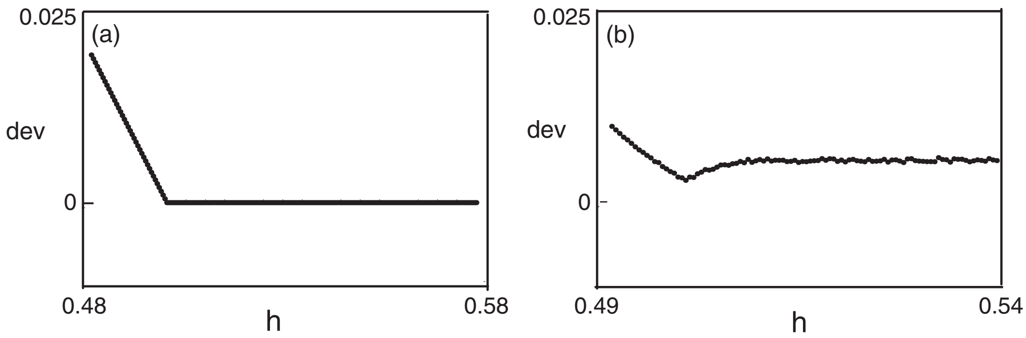

Figure 4. Dependence of dev on the control point h (summation over 500 orbit points, period-one orbit). (a) For zero noise, a piecewise linear function with a minimum (= the optimal noise-free control point) emerges. (b) In the presence of noise (str = 0.02), the function becomes nonlinear, with a nonzero minimum at the optimal noise-free control point.

control goal would then be to control the system as closely as possible along the underlying noise-free system. In the numerical control results presented below, we deal with both control goals.

5. Control results

To measure the efficacy in performing control on natural cycles, we define the control distance as the absolute difference between the 'natural' underlying solution and the controlled solution, per step. If the underlying system is of periodicity larger than one and the control is on a period-one fixed point, the control distance becomes particularly large. If the underlying system is in the chaotic regime, all cycles are unstable, and the control of any of them is *a priori* equally well justified and natural. In particular, control can be established on a period-one orbit with zero control distance in the zero-noise limit. Below, we discuss the most salient numerical results obtained from applying HLC to noisy systems.

5.1. Control in the stable system regime

For our numerical investigations, we restrict ourselves to the control of superstable orbits (by choosing $a = 2$ and $1 + 5^{1/2}$, for the periods one and two, respectively), and apply the control at the cycle maximum. As a measure of efficacy, we calculate the average deviation of the noisy control relative to the noise-free system, denoted by dev, as a function of the noise and of the limiter position h. This seems to reflect best the natural tendency of the system to return to the vicinity of the uncontrolled noise-free system once the control is relaxed. We find that for zero noise, dev(h) is a piecewise linear function (shown in figure 4(a) for the period-one orbit), where the nonzero slope, associated with h below the maximum of the function f, is determined by the periodicity and by the amount of nonlinearity expressed by a. For nonzero noise, the formerly piecewise linear function becomes nonlinear, with the minimum being situated at the optimal control point of the noise-free system. For noise strengths str < 0.1, which we consider to be a realistic

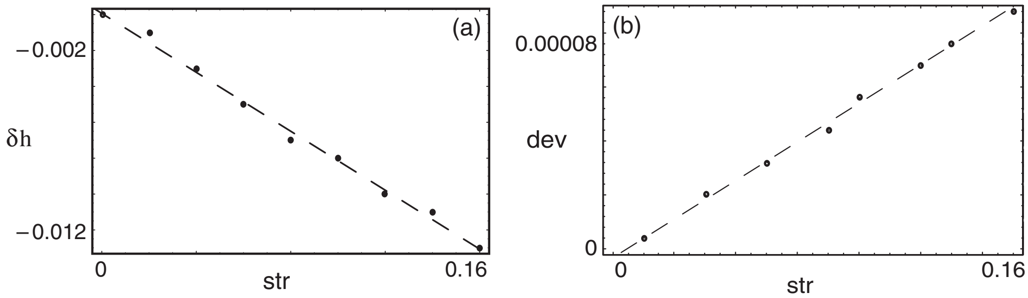

Figure 5. Results for the chaotic regime, where an unstable period-two orbit is controlled. (a) Linear dependence of the optimal control point displacement δh on the noise strength str. (b) Linear dependence of dev at the optimal control point on the noise strength str.

case, the deviation is a linear function of str (see figure 4(b)). The stronger the required corrections, the more the orbit histograms focus around the control point. The controlled orbit, however, deviates ever more from the original system orbit, which leads to a fast increase of dev. The control of the superstable period-two orbit yields a similar picture. One important difference, though, is that the amount of noise allowing maintenance of the control decreases considerably. For stable orbits, control can beneficially be applied up to relatively large noise levels (str $\sim$ 0.08). Control is lost when, due to the noise, interchange of orbit points occurs. This is why in the presence of a substantial amount of noise, only low-order cycles can be controlled. For a period-four orbit, a noise level of str $>$ 0.01 already leads to control loss. If the correct ordering of the cycle points has no importance, control beyond orbit point interchange will be beneficial. Interestingly, the function $\text{dev}(h, \text{str})$ scales linearly with str (identical curves emerge if h and dev are replaced by h/str and dev/str, respectively). As a rule of thumb, by means of optimal control, the deviation can be reduced by a factor of $\sim$0.5.

5.2. Control in the chaotic system regime

To investigate the control in the chaotic regime, we focus on the fully developed logistic map ($a = 4$). To control true system orbits, the control point must be chosen at locations corresponding to the bifurcation points (see figure 3(a)), whose location can be evaluated analytically [25]. Without control, chaos prevents the system from staying on a given cycle. As a consequence, the efficacy of the control is measured as the difference between controlled noise-free and controlled noisy systems. In order to obtain a period-one orbit in the noise-free case, the limiter was adjusted to $h = 0.75$. Experiments show that in the presence of noise, the optimal control point moves away from the noise-free optimal control point. This is in contrast to the behaviour in the stable regime, and may help to distinguish between the two cases. The displacement is a linear function of the noise strength, as is the deviation dev measured at the optimal shifted control point. To provide an unstable noise-free period-two orbit, the controller was adjusted at $h = 0.904$. Again, the optimal control point's displacement and the minimal deviation are linear in the noise strength (see figures 5(a), (b)). Controlling period-four orbits yields an even stronger shift from the optimal noise-free controller position at $h = 0.925$. The amount of sustainable noise, however, is further reduced if compared to the period-two case (by a factor of ~ 0.5). Beyond a noise strength of str $= 0.04$, the orbit escapes control.

6. Conclusions

Control mechanisms of limiter type are common in economics. This control, however, inherently generates superstable system behaviour, whether the underlying behaviour be periodic or chaotic. *A priori*, a frequent change of the position of the limiter might appear to be a suitable strategy in order to compensate for the amplified or newly created cyclic behaviour. This strategy, however, will only result in ever more erratic system behaviour. Our analysis shows that it is advantageous to keep the limiter fixed, adjusting it only over timescales where the system parameter a changes noticeably. In this way, reliable cycles of smaller periodicity will emerge. Among these cycles, the period-one cycle appears to be the optimal one, from most economic points of view. To recruit this state, a strong initial intervention is necessary and the control must be permanent. Otherwise, a strong relaxation onto the suboptimal natural behaviour sets in. In discussions of real economics, these effects will be natural arguments against the proposed control. To overcome such arguments, a sufficiently simple control policy must be formulated in democratic societies. The framework discussed may provide the basis for the formulation of a control policy for attaining economic optimality.

As the most obvious challenge to the proposed control strategy, instead of maximizing the average $\bar{x}$, the minimal distance to the noise-free dynamics could be chosen as the control target. We demonstrated that when a varies slowly, this control generally does not lead to an optimum of $\bar{x}$. If superstable orbits are controlled at the highest orbit point of the noise-free behaviour, the location of the optimal control point is independent from the noise strength. In the chaotic regime, in contrast, the optimal control point is displaced, linearly in the noise strength. Controlling at this point reduces the dev error by roughly one fourth, if compared to the control at the noise-free optimal point. Detailed investigations show that the observed shift of the optimal control point also depends on the nature of the noise. If purely positive noise is added, the shift vanishes. From the perspective of economics, HLC-induced noise reduction can be regarded as a substantial improvement. However, since the period-one orbit has substantial advantages, the control of the latter state is preferable.

Our framework could be of relevance for better understanding and monitoring of economic behaviours. It has been found [29] that for either very underdeveloped or developed economies, stable fixed-point behaviour is predominant. At an intermediate level, however, complex economics emerges that can induce chaotic dynamics of the entrepreneur's wealth, W_n [30]. In order to control this case, HLC in the form a tax on assets with a sufficiently fast progression could be applied, forcing W_n to remain below a maximal value, $W_{\max}$. With sufficient care, HLC on a period-one system could be achieved, and excessive economic variations due to chaotic dynamics could be prevented. Political realizability will often require the use of 'softer' limiters (in the sense that $W_n > W_{\max}$ is not strictly prohibited), but the main features of HLC will be valid even in these cases [26].

We emphasize that short-term cycles emerge at all levels of economics. It has become, e.g., a common observation that the demands for certain professionals (in central Europe in particular for teachers) undergo large fluctuations from one year to another. In one year, severe problems are encountered in recruiting a sufficient number, so that the professional requirements have to be lowered, whereas in the next year, there is an excess supply. We propose to interpret this as the signature of an economy that has moved out of period-one behaviour. From the teaching quality as well as from the individuals' biographies points of view, the occurrence of this effect should be prevented or smoothed. Our approach offers a perspective for understanding, studying and, potentially, also engineering such phenomena.

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