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Nonlinear analysis of epileptic activity in rabbit neocortex

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Abstract. We report on the nonlinear analysis of electroencephalogram (EEG) recordings in the rabbit visual cortex. Epileptic seizures were induced by local penicillin application and triggered by visual stimulation. The analysis procedures for nonlinear signals have been developed over the past few years and applied primarily to physical systems. This is an early application to biological systems and the first to EEG data. We find that during epileptic activity, both global and local embedding dimensions are reduced with respect to nonepileptic activity. Interestingly, these values are very low ($d_E \approx 3$) and do not change between preictal and tonic stages of epileptic activity, also the Lyapunov dimension remains constant. However, between these two stages the manifestations of the local dynamics change quite drastically, as can be seen, e.g., from the shape of the attractors. Furthermore, the largest Lyapunov exponent is reduced by a factor of about two in the second stage and characterizes the difference in dynamics. Thus, the occurrence of clinical symptoms associated with the tonic seizure activity seems to be mainly related to the local dynamics of the nonlinear system. These results thus seem to give a strong indication that the dynamics remains much the same in these stages of behavior, and changes are due to alterations in model parameters and consequent bifurcations of the observed orbits.

1 Introduction

Tools for the analysis of signals from nonlinear sources have improved in recent years, allowing the study of observations from nonlinear biological and physical systems to be treated in a quantitative manner (Abarbanel 1996). When such systems have three or more degrees of freedom, they can exhibit chaotic motions which are spectrally continuous and broadband. Such signals do not yield to familiar Fourier-based analysis, but using the ideas of attractors in time domain and classification of systems by properties of these attractors, one can make predictive models of such systems and, in many cases, when the freedom to vary some external parameters is possible, even control them to regular motions.

The description of nonlinear systems which exhibit chaos is properly pursued in a multidimensional space where the attractor is unfolded from its projection on the axis of measurements. To perform this unfolding, one uses methods described below to establish a global coordinate system whose (integer) dimension d_E provides us with a clear sense of the total number of degrees of freedom operating to produce the measured signals and a sense of the complexity of the dynamics. The dynamics itself is characterized by a local (integer) dimension $d_L \leq d_E$ which tells us locally how many differential equations or discrete time maps are needed to capture the dynamic development of the source of our signals. With these pieces of information, one can evaluate, from observed data alone, the stability indices called Lyapunov exponents which characterize how rapidly nearby orbits on the attractor diverge or converge in time. The largest exponent λ_1, if it is positive, tells us that the system is chaotic and provides a quantitative measure of the predictability of the observed system. Predictions based on observations or underlying dynamic models cannot be accurate for periods much longer than the order of $1/\lambda_1$. In this paper, we apply a number of these tools to the understanding of electroencephalogram (EEG) activity induced in the rabbit visual cortex.

The application of concepts from this theory to EEG data has provided a number of interesting insights. There is evidence that EEG signals can be interpreted as the output of a deterministic system of relatively low complexity, containing highly nonlinear elements. Since the interaction among neurons is nonlinear, perhaps part of this is not surprising, but the enormous number of neurons involved in generating macroscopic voltage recordings makes low complexity less obvious. There are examples both from biological and physical systems in which one sees such results, and typically the measurements involve an average of some kind over many inputs and over short time scales. Indeed, this may well lie behind such observations in EEG analyses. Several authors have used nonlinear methods to analyze EEG activity of the brain in various states, for example (Skarda and Freeman 1987; Fell et al. 1993;

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Müller-Gerking et al. 1996). In these and similar studies, however, the application of nonlinear analysis algorithms does not always lead to the conclusion of low-dimensional dynamics as was first expected. It is thus still not clear that EEG is really a chaotic signal. If so, the questions still remain of whether the methods of analysis at hand are useful instruments to describe the data and whether we can use our knowledge of dynamic systems theory to understand some principles of cortical functioning. The methods we employ in this study are not so sensitive to errors or noise in the measurements, and they have proven robust in allowing the evaluation of Lyapunov exponents and the creation of predictive models of complex phenomena.

In our approach to these issues, we have chosen to analyze the EEG of epileptic activity which constitutes an interesting example of brain activity, not the least because here there is still evidence for chaos. Furthermore, increased understanding of the mechanisms of epilepsy promises to be important for clinical practice. This is why a variety of studies have investigated the nonlinear properties of epileptic seizure activity either in experimental animals (Pijn et al. 1991; Beldhuis et al. 1993) or in clinical data (Babloyantz and Destexhe 1986; Iasemidis et al. 1990; Lehnertz and Elger 1995; Theiler 1995; Lerner 1996).

As an example, we present in Fig. 1 the EEG measured in the visual cortex of a rabbit during stages of epileptic activity. According to the current model of epileptogenesis in the neocortex, an epileptic focus is surrounded by a massive inhibitory zone, which prevents the further spread of excitation to adjacent neurons. While the focus generates single preictal spikes by massive depolarization of neurons, the surrounding neurons generate long-lasting inhibitory potentials. During the transition into the ictal phase, the inhibitory zone becomes partially weakened, so that the spread of excitation from within the focus is facilitated. Due to the spread of excitation along anatomical interneuronal connections, larger areas of cortical tissue become involved in the seizure, and rhythmic potential oscillations appear in the EEG. Typically, these oscillations have frequencies around 20-30 Hz during the initial phase, that is, in the first few seconds. During the so-called tonic phase, measured potentials oscillate almost sinusoidally around 7-12 Hz, whereas clonic oscillations are less regular. In general, these different states of activity in epileptogenic tissue can be clearly distinguished by electrographic features. We have chosen the data in Fig. 1 as an example because the transition from preictal to tonic activity is clearly visible, and this transition often also marks the onset of clinical symptoms.

In this paper, we report the analysis of EEG with a set of algorithms that have recently been developed (Abarbanel et al. 1993; Abarbanel 1996). These algorithms concentrate on carefully constructing a working state space for the source of our measurements using the measured voltages and their time delays. In this space, a global dimension – number of independent coordinates – is established which allows the unambiguous determination of properties of the system attractor. Local aspects of the dynamics on the attractor are also identified and used in the analysis. We find that during epileptic activity, both global and local embedding dimensions are reduced with respect to nonepileptic activity. These integer dimensions characterize the level of excitation of a system in that they are directly related to the number of observed degrees of freedom in the signal. Interestingly, these values do not change between preictal and tonic stages of epileptic activity. However, between two stages the local dynamics change quite drastically, as can be seen, e.g., from the shape of the attractors. Furthermore, the largest Lyapunov exponent λ_1 is reduced by a factor of about two at the transition from preictal to tonic stages of activity. A factor of two in the exponent which governs the exponential loss of information or predictability is a significant change. In both stages the largest Lyapunov exponent is positive, thus proving the presence of chaos in the system.

2 Experimental

Experiments were performed in rabbits of either sex with an average weight of 3 kg. All experimental procedures comply with the Principles of Laboratory Animal Care (NIH Publication No. 86-23). The rabbits were tracheotomized and intubated under short-term barbiturate anesthesia. Thereafter, animals were placed in a stereotactic frame and artificially ventilated under muscle relaxation (Alloferin 0.5 mg/h). Then anesthesia was continued with Fluothane (2 vol%). During the entire experiment, heart rate and CO_2 end tidal concentration were monitored to ensure proper conditions. After trepanation of the skull overlying the visual cortex, the dura was carefully removed and the neocortex covered with prewarmed CSF-agar (4%). This microelectrode array was inserted perpendicular to the cortical surface. The device consisted of 16 contacts in a row at spacings of 125 μm (Petsche et al. 1984). This setup allowed the simultaneous recording of EEG from within each neocortical layer. After a 30-min pause during which the neocortical activity stabilized at a typical background pattern of spontaneous activity under constant anesthesia, recordings and stimulation procedures were commenced. As stimulation, we projected a flickering random dot pattern via an optical fiber system to the contralateral eye. Focal epileptiform activity was induced by local application of a 170 mM solution of penicillin to the neocortical surface. The total fluid volume was 1 μl. As described previously, such a procedure leads to the development of epileptiform discharges (preictal spikes by definition) within several minutes, and ictal discharges (the electrical phenomena during clinical seizures) after approximately 60 min (Pockberger et al. 1984). All data recordings were stored on analog tape with bandwidth 0-1250 Hz and digitized off-line at 1024 samples/s. In this paper, we will only present and discuss data from 1 of the 16 channels. This channel was located in layer V of the neocortex and showed the highest signal-to-noise ratio in all experiments.

3 Method of analysis: stages of epileptic activity

In this section, we guide the reader through the steps of the analysis. The data presented in Fig. 1 are a sequence of voltage measurements $v(t_i)$ sampled at discrete time points $t_i = t_0, t_1, ..., t_m$. The goals of nonlinear dynamical analysis are to first establish a multivariate phase space in which the attractor of the dynamics is unfolded from its projection on the measured voltage axis. Next, one analyzes

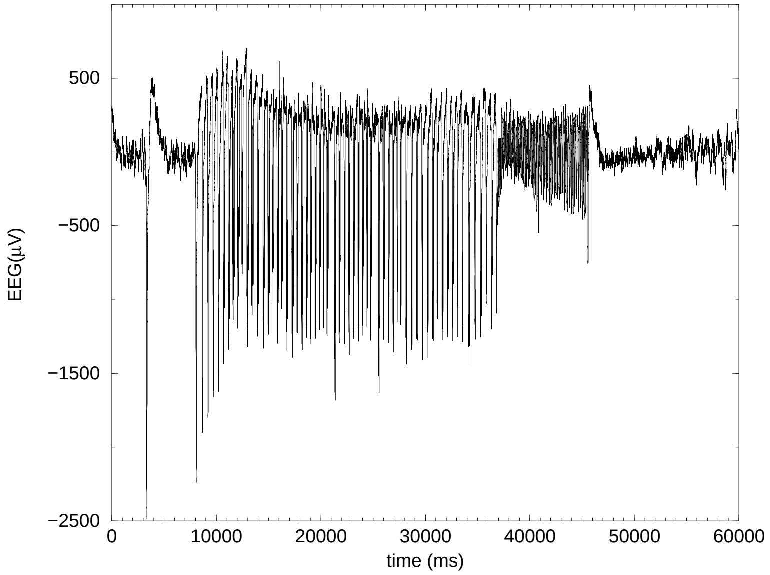

Fig. 1. Electroencephalogram (*EEG*) from layer V of the rabbit's visual cortex (area OC1). During the preictal phase, the epileptic focus generated spikes, i.e. potential transients with a duration of less than 70 ms. Usually, such transients as, e.g., the first spike at 3000 ms occur spontaneously at more or less regular intervals of 1-2 s. Later in our experiment (starting from 8000 ms), dense preictal spikes were triggered by a flickerlight stimulation of 1 Hz. Rather suddenly, this spiking activity was succeeded by epileptic seizure activity (the ictal phase), which is characterized by patterned potential oscillations, in this case a tonic seizure. Finally, the seizure abruptly stopped and was followed by postictal voltage depression. In this paper, we focus on the two experimental stages 'dense preictal spikes' and 'tonic seizure'

the vector data in the coordinate system established this way. The coordinates of the phase space are the voltage $v(n) = v(t_0 + n\tau_s)$ measured every τ_s seconds starting from some time t_0 and the $d_E - 1$ time delays of this voltage $v(n+Tk),\ k = 0, 1, 2, d_E - 1$ with the time lag T an integer indicating the multiple of τ_s used in constructing the data vectors. The vectors made this way

$$\mathbf{y}(n) = [v(n), v(n+T), v(n+2T), \ldots, v(n+(d_E-1)T)] \tag{1}$$

replace the original scalar data $v(n)$ in defining the system. The vectors $\mathbf{y}(n)$ of our example are shown in Fig. 2 for preictal spikes (top) and tonic seizure activity (bottom). The parameters d_E= 2 and T = 20 ms were chosen to give a clear visualization of the attractors. As one can see in this figure, the attractor of the tonic phase is rather cyclic, and the preictal spikes generate a somewhat more complex attractor.

In our analysis, we first determine the time lag T by asking how much information is learned about the measurement $v(n + kT)$ from the measurement $v(n + (k-1)T)$ on average over all data. This is a kind of nonlinear correlation function because the definition of information requires the joint distribution of the two measurements. It is called average mutual information (AMI) and can be normalized so that AMI(T = 0) = 1. In Fig. 3 we show the AMI for the two stages of the experiment. As expected, the curve for preictal spikes is rather smooth, whereas there is oscillatory structure for the tonic seizure. The time lag where the AMI falls to 20% determines a practical value for T. This criterion leads us to about T = 20 ms for both stages of epileptic activity.

We begin our search for characteristics of the dynamics by evaluating the global dimension d_E required to unfold the attractor for the source of these data. There are several ways to calculate estimates for this dimension. One well established approach (Grassberger and Procaccia 1983) analyzes the static distribution of the points on the attractor and yields the correlation dimension d_2. For our data, this algorithm leads to $d_2 < 3$ for the tonic seizure activity (Fig. 4). In the preictal phase, we do not find a plateau, indicating that d_2 is either very large or that the applied algorithm

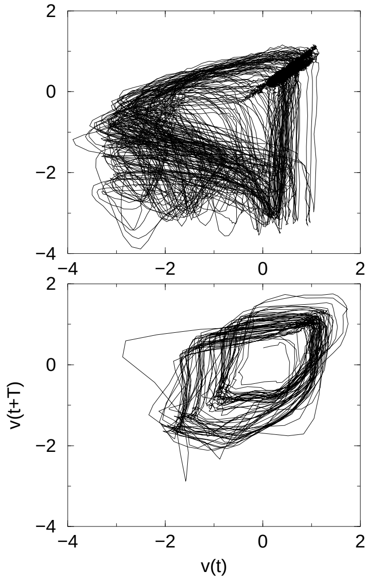

Fig. 2. EEG data during preictal spikes (*top*) and a tonic seizure (*bottom*) embedded in a two-dimensional phase space $(v(t), v(t+T)), T = 20$ ms. Units are standard deviations around the mean at (0,0). Orbits have 54 (*top*) and 50 (*bottom*) cycles

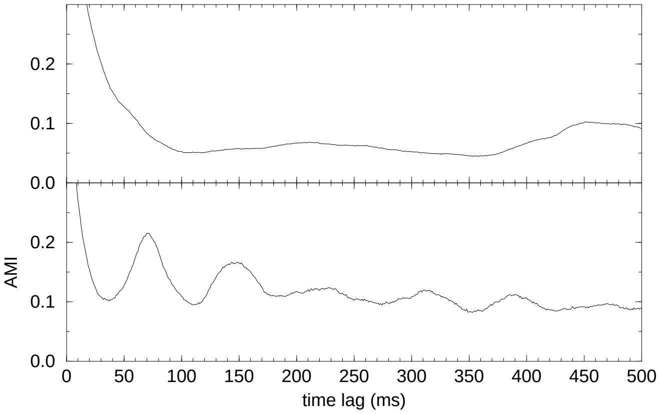

Fig. 3. Average mutual information (*Ami*) calculated for EEG data taken during preictal spikes (*top*) and tonic seizure (*bottom*)

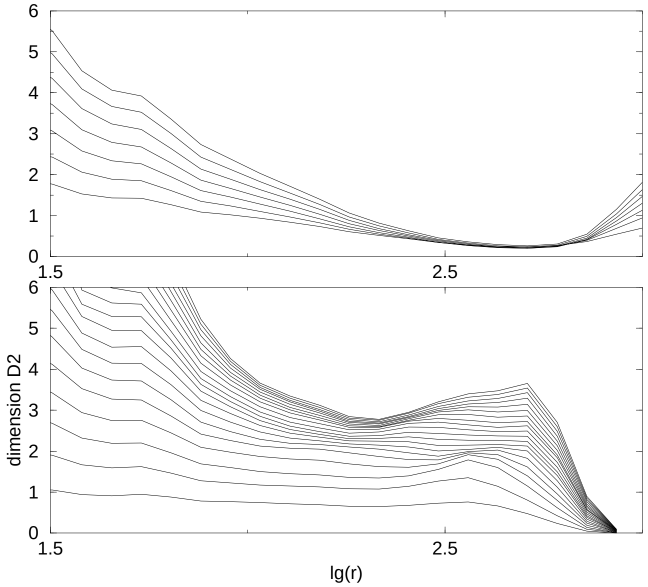

Fig. 4. Correlation dimension d_2 estimated for preictal spikes (*top*) and tonic seizure (*bottom*). A plateau was reached in the tonic case, indicating a correlation dimension $d_2 < 3$. For the case of preictal spikes, we could not produce a plateau

does not yield reasonable results with our biological data. As a conceptual disadvantage, the correlation dimension describes how the sample of points along a system orbit tend to be distributed spatially, but there is no information about the dynamic, temporally evolving structure of the system.

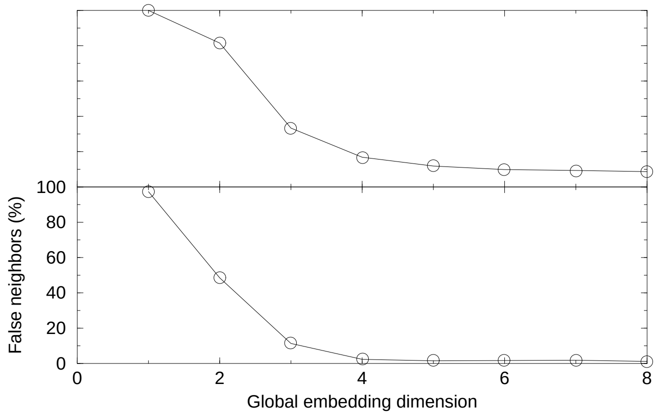

Fig. 5. Percentage of global false nearest neighbors as a function of embedding dimension for EEG data during preictal (*top*) and a tonic (*bottom*) activity

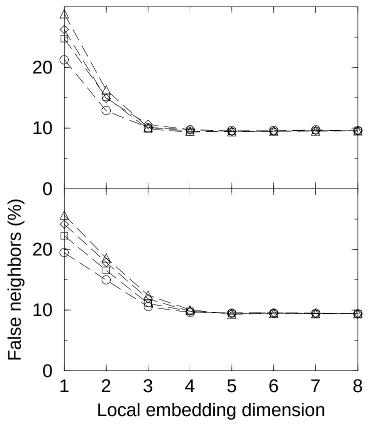

Fig. 6. Percentage of false nearest neighbors as a function of local embedding dimension for EEG data during preictal (*top*) and a tonic (*bottom*) activity

Another method directly asks the question: when has one eliminated false crossings of the orbit with itself because the attractor was projected into a too low dimensional space? This dimension is determined by asking in which dimension the orbits **y**(n), $n = 1, 2, ...$ no longer intersect each other because of the projection from high dimension to dimension $d = 1, 2, ..., d_E$. This lack of overlap is established by looking at the nearest neighbor of each point **y**(n) in dimension $d = 1, 2, ...$ and asking whether that neighbor remains a neighbor in dimension $d + 1$. If all nearest neighbors remain neighbors as the dimension is increased, then all points are true neighbors rather than being false neighbors due to projection from a higher dimension. This false nearest neighbors test (Abarbanel et al. 1993; Abarbanel 1996) works robustly even in the presence of noise identifying when a signal comes from a low-dimensional system. For our data (Fig. 5), the percentage of false nearest neighbors is independent of embedding dimension for $d \geq 5$ in both cases. In the case of the tonic seizure, less than 5% false nearest neighbors remains at higher dimensions. We qualify this part of the signal as noise, meaning completely unpredictable and random. In the preictal phase, this noise amounts to about 10%. This might explain why no correlation dimension d_2 was measurable in this phase, as mentioned above. Using the false nearest neighbor algorithm, we find a global dimension of about $d_E = 5$ for both preictal phase and tonic seizure.

The dynamics of the system may evolve in a dimension lower than the global dimension required to unfold an attractor. The local dimension of the dynamics d_L is established by going onto the attractor, as unfolded globally in dimension d_E, and asking what dimension is required locally to produce a good map from the neighborhood of a point **y**(n) to the neighborhood of the point **y**$(n{+}1)$ on average over the attractor. When the quality of prediction becomes independent of the number of neighbors N_B and the dimension of the local map, we have located d_L (Abarbanel and Kennel 1993). For the preictal stage we see in Fig. 6 (top) that this occurs at $d_L = 3$. For the tonic seizure (bottom), this occurs at $d_L = 4$, telling us that a 4-dimensional dynamic system will capture all the information in the recording during this condition.

Finally, in recounting the analysis of the intracortical recordings, we report the local Lyapunov exponents associated with the dynamics of the system (fig 7). They are invariants associated with the dynamical evolution on the attractor, and they give a sense of the instability associated with the formation of the attractor, as well as of the ability to make predictive models of the nonlinear system. In this calculation, we rely on the values for the parameters T, d_E, and d_L as estimated above. The criteria for validity of this calculation are: (i) the sum of the exponents must be less than zero; (ii) one exponent must approach zero for great lengths L; (iii) forward and backward calculations must give the same results. The first of these assures us that the system

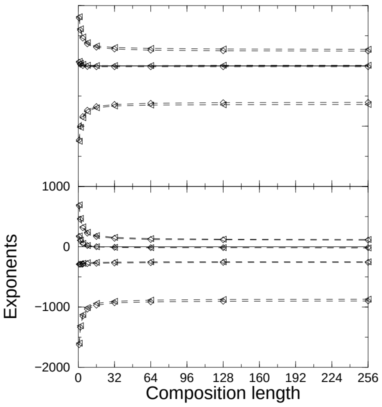

Fig. 7. Average local Lyapunov exponents calculated from the EEG data. For the preictal (tonic) stage 23000 (8000) data points were used. The embedding dimension was d_E= 5, and the order of local polynomials was taken to be d_L= 3 (d_L= 4). In all, 6000 initial conditions were used.

as a whole is stable and volumes in state space contract. The absence of this property would indicate a flaw in our data, as such a system would blow up in time. The second property tells us that ordinary differential equations need to be followed to describe these data. The final condition assures us that each of the exponents is connected with dynamics. False exponents do not exactly reverse sign when time is reversed, so we could identify incorrect local dimensions with this computation.

The largest of the Lyapunov exponents (λ_1) is positive for both experimental conditions, which is strong evidence for the presence of chaos in our intracortical recordings. λ_1 measures how quickly linear distances grow. Two points initially separated by an infinitesimal distance ϵ will, on average, separate as $\epsilon e^{\lambda_1 \cdot t}$. Conversely, one may deduce a prediction horizon $t_H = 1/\lambda_1$ in units of milliseconds. Its value t_H = 16 ms for the flicker condition is only about half as long as during the tonic seizure (t_H = 35 ms) in this experiment. The exponents can be used to calculate the Lyapunov dimension of the system according to

$$d_\lambda = K + \frac{\sum_{i=1}^{K} \lambda_i}{|\lambda_{K+1}|} \tag{2}$$

where $\sum_{i=1}^{K} \lambda_i > 0$ while $\sum_{i=1}^{K+1} \lambda_i < 0$. This is a fractal dimension expected to be similar in numerical value to the d_2 discussed above. We obtain the value d_λ= 2.4 for both experimental conditions. This is further evidence that the system can be described by low-dimensional dynamics.

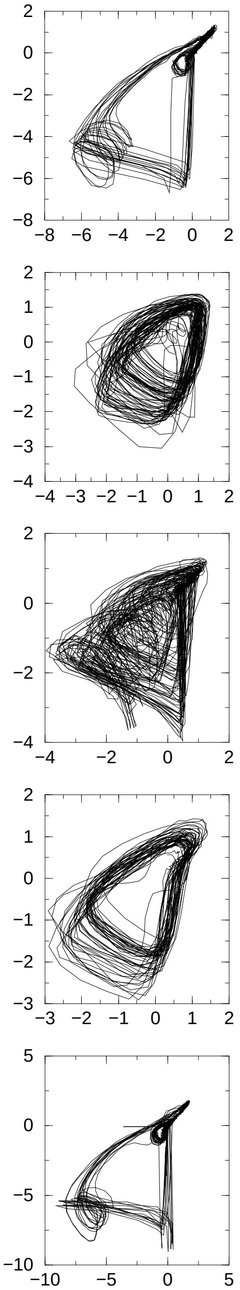

Fig. 8. EEG data embedded in a two-dimensional phase space $(v(t), v(t + T)), T$ = 20 ms. The stages of activity are (from *top* to *bottom*) preictal spikes, tonic seizure, clonic activity, again tonic seizure, postictal spikes. Units are standard deviations around the mean at (0,0)

4 Analysis of other, similar epileptic activity

In similar experiments, the same main features could be reproduced. For example, some attractors are show in Fig. 8. The corresponding data for the different stages of epileptic activity was recorded in one continuing experiment with short transition intervals. In the top graph, visual stimulation elicited spikes until a tonic seizure emerged for a few seconds. The tonic seizure was followed by clonic activity. The system then returned to a second tonic seizure. Finally, postictal spikes occurred over several minutes. During the tonic phase, the attractors are rather cyclic. The attractors during preictal and postictal spikes have more structure and are very similar to each other. Note also the similarity to Fig. 2.

To compare the results of our analysis for all experiments, we have grouped the values in Table 1. For both experimental conditions, preictal spikes and tonic seizure, the global dimension d_E is about 5. The local dimension d_L is somewhat lower, but again similar for both conditions. The same is also true for the Lyapunov dimension d_λ. The only consistent differences emerge for the prediction hori-

Table 1. Dimensions and prediction horizon $t_H = 1/\lambda_1$ of the EEG during epileptic activity

Experiment	T (ms)	d_E	d_L	d_λ	t_H (ms)
Triggered preictal spikes					
401ep2	20	5	3		
404ep1	20	5	3	2.69	10
404ep2	20	5	4	2.66	53
406ep3	20	5	4-5		
406ep7	20	5	3		
428fl	20	5	3	2.38	16
Tonic seizure					
401ton	20	5-6	4-6	2.32	60
404ep4	20	5	4	2.24	36
406epn	20	5	4	2.46	35
406ep9	20	5	3-4	2.31	12
410ep1	20	5	4	2.69	41
428ton	20	5	4	2.37	35

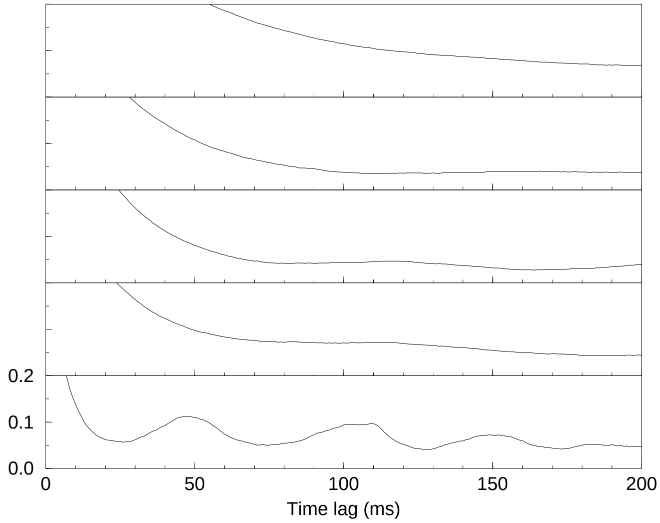

Fig. 9. Ami for the five experimental conditions (from *top* to *bottom*) spontaneous activity; flickerlight stimulation with 0.5 Hz, 5 Hz, 9 Hz; and tonic seizure

zons $t_H = 1/\lambda_1$, which are in general higher during the more regular, tonic activity.

5 Analysis of nonepileptic activity

As before, in this experiment, the EEG was recorded from layer V in the visual cortex of an anesthetized rabbit. However, the cortex was only prepared with an epileptogenic substance in the last of the five experimental conditions. Figure 9 shows the AMI for (a) spontaneous activity; during flickerlight stimulation with (b) 0.5 Hz, (c) 5 Hz, (d) 9 Hz; and (e) during the tonic phase of a seizure after application of penicillin. First note that the highest overall curve is found for the spontaneous activity. This indicates that the system is engaged in some concerted idling activity and evolving undisturbedly. During flickerlight stimulation, AMI is reduced, and oscillatory structure appears. The oscillatory behavior of the AMI becomes more pronounced with increasing frequency of the flickerlight stimulation and is strongest during the epileptic seizure.

In Table 2 we list the global and local embedding dimensions for the five experimental conditions. The dimensionality is highest for the resting state, despite the high AMI. The values are smallest during the seizure.

Table 2. Dimensions and prediction horizon of the EEG during nonepileptic activity

condition	T (ms)	d_E	d_L	d_λ	t_H (ms)
Spontaneous	52	>12	>12		
0.5 Hz flicker	24	6	4		
5 Hz flicker	24	6	4		
9 Hz flicker	24	6	4		
tonic seizure	20	5	4	2.69	41

6 Discussion

Our results show that EEG signals of epileptogenic activity fulfill all requirements to be characterized as low-dimensional chaos, since the global and local embedding dimensions are low, and the largest Lyapunov exponent λ_1 is positive. Our analysis differs from earlier work primarily in stressing those aspects of the dynamics of the source which are connected with prediction and model building. While we did evaluate a fractal dimension using the Lyapunov dimension, this in itself is only one piece of the overall set of tasks in which one is interested when presented with data such as ours. In this paper, we discussed only the determination of an appropriate state space in which to analyze our data and evaluated the global Lyapunov exponents to assure ourselves that we did have chaos, to determine the predictability horizon for the EEG processes and to check that d_λ is consistent with other determinations of fractal dimension. We will return to model construction and comparison of predictions with other aspects of these experiments in our further work.

At this point, it seems necessary to discuss the validity of determining the value of a parameter like d_E or λ_1 which, in theory, is only defined for stationary processes. In the complex biological system of interest, it is clear that only finite stretches can be singled out that may not strictly comply with the standards for stationarity as postulated by nonlinear systems theory. This is definitely true for the calculation of the Lyapunov spectrum, which could require a somewhat larger number of cycles of the attractor in order to satisfy the requirement of ergodicity. In practice, we can never satisfy the formal, rigorous mathematical requirements associated with ergodicity and the multiplicative ergodic theorem which underlies the evaluation of Lyapunov exponents. As a rule, however, it is apparent that the number of neighbors used to make the local maps utilized for evaluation of local Jacobians of the underlying dynamics is what is critical. As the system moves around its attractor, the neighborhoods of each point become more and more populated. When a ‘small’ neighborhood of a point (say on the order of 1% of the size of the attractor) has enough neighbors to safely determine the local Jacobians, we may be assured that our estimate of the Lyapunov exponents is on a sound footing. For further details of this argument, see Abarbanel (1996). The calculations presented in this paper meet the criteria mentioned earlier (negative sum of exponents, one exponent approaches zero, forward and backward calculations give same results). Therefore, it seems that we have enough data in nearly stationary domains that our estimated exponents can be treated with some confidence; we cannot give a formal bound on this, but have found the estimates to be repeatable and somewhat independent of the choices for characterizing the embedding space. We, therefore, have to view the calculated values as estimates, which nevertheless gain their significance through the reproducibility in several experiments. In the case of low embedding dimensions ($<$ 4), the two-dimensional representation of the attractors (Fig. 2) also makes the calculated values seem reasonable. These absolute values may thus serve as an estimate of the type of dynamics generated by the underlying neuronal system. Still, in a biological system, these values have to be viewed with caution, in particular for higher dimensions and also for the value of λ_1. We therefore propose a qualitative approach where merely the differences of values determined for different stages should help to characterize the complex temporal dynamics of the epileptogenic activity.

It is usually assumed in the literature that there exists a mathematically well-defined decrease in complexity associated with an epileptic seizure which in some sense reflects what is evident to the naked eye. Along this line of thought, several groups have demonstrated that some embedding dimension or the correlation dimension d_2 decreases before or during the onset of a seizure (Skarda and Freeman 1987; Pijn et al. 1991; Lehnertz and Elger 1995). It was therefore not surprising that we would find global and local embedding dimensions to be smaller in the epileptic case than in the case of spontaneous activity. As an aside, the dimension related to flickerlight-driven activity of the visual cortex lies in between the two extreme cases. However, the embedding dimensions turned out to be essentially the same for preictal spiking activity and tonic seizure activity.

Here it is interesting to note that the dimensionality of the system does not change, even though preictal and ictal stage are clearly distinguishable from the voltage traces and from a clinical point of view in that epileptic symptoms only occur during tonic or clonic seizure activity. However, a considerable change takes place in the local dynamics as can be judged from the attractor-plots. This is potentially quite good news from the point of view of modeling the EEG processes involved here. If the model is basically the same in each domain but the parameters of the model have altered, our task in identifying good models is much simplified. We need not seek different functional forms or numbers of degrees of freedom for different macroscopic regimes, but instead can seek a bifurcation-like behavior of one underlying model. Furthermore, we found the highest local Lyapunov exponent λ_1 related to tonic seizure activity to be lower than during preictal spikes by a factor of about two. This means that the calculation of the local Lyapunov spectrum allows us to distinguish the two stages of epilepsy. In a related work by Iasemidis et al. (1990), it was shown that by calculating only λ_1, the transition from resting activity to seizure activity could be monitored. From a physical point of view, a drop of λ_1 by a factor of 2 indicates a largely increased predictability of the dynamics of the system during the seizure. Our calculation of the whole spectrum of Lyapunov exponents gives us the further possibility to calculate the Lyapunov dimension d_λ. In accordance with the other two measures of dimension d_E and d_L, d_λ also yields the same value for preictal spikes and tonic activity. Since d_λ depends on the sum of many λ_i (Eq. 2), this means that lower exponents compensate for the change of the largest component λ_1. The following picture emerges: the dynamics of the EEG system seems to be restricted to about 3 dimensions; even a change in the epileptic state, i.e. a drastic change in the dynamics, does not change this dimensionality. Thus, the occurrence of clinical symptoms during the tonic stage seems to be mainly related to the local dynamics of the nonlinear system, and as noted above, representing this dynamics in various regimes appears much more approachable than one might have surmised *a priori*.

In this paper, we have restricted ourselves to investigating the validity of nonlinear systems theory to EEG data recorded from always the same site and depth in the neocortex. That is, of course only a first step in understanding the relevance of this theory to epilepsy, since epileptic activity is typically characterized by an interaction of neurons spread out over significant distances (Petsche et al. 1984; Pockberger et al. 1984). To begin with, preictal spikes are generated by distinct spatio-temporal patterns of current sources and sinks distributed within different layers in one cortical column. The transition to ictal behavior is then thought to result from an interaction between several cortical columns which are located a few millimeters apart. In this way, several subfoci entertain the oscillations in electrical potential as they are observed in the course of tonic or clonic seizures. Future work will therefore also analyze this spatial scale of epileptic activity.

In conclusion, we have presented a comprehensive investigation of embedding dimensions and local dynamics on a data set of EEG during epileptiform activity in the neocortex. Our consistent finding of low-dimensional attractors with $d_E \approx 3$ suggests that the underlying processes may be simple enough to be identifiable. The dynamics during preictal and tonic stages of epileptic activity seem to remain much the same, and changes of the system's behavior are due to bifurcations of the orbit. On the basis of nonlinear systems theory, powerful techniques have been developed to control chaos in the hippocampus by minimal electrical stimulation (Schiff et al. 1994). Integrating nonlinear systems theory also into a concept of basic mechanisms of epilepsy in the neocortex thus seems very promising, both scientifically and clinically.

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